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Active Impedance Matching for Superdirective, Super-Gain HTS Antenna Arrays

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Naval Air Warfare Center Weapons Division

FOREWORD

This report presents the electromagnetic circuit of a colocated electrically small dipole and loop antenna employing feedback matching. This work was performed at the Naval Air Warfare Center Weapons Division, China Lake, Calif., during fiscal year 1994 in support of an Accelerated Technology Initiative investigating High-Temperature Superconducting Antennas sponsored by the Office of Naval Research, Information, Electronics and Surveillance Science and Technology Department (ONR31). This work was monitored by Dr. Donald H. Liebenberg under fund document N0001495WX20154.

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13. ABSTRACT *(Maximum 200 words)*

(U) The radiation pattern of a super-gain antenna array is determined by specifying the antenna element currents in both magnitude and phase. The spacing between elements in such an array is small with respect to a wavelength and interelement coupling cannot be ignored. The specification of the element currents together with this coupling produces an active input impedance to each element that may be very different from the usual antenna impedance of the individual elements.

(U) For those active impedances with a positive real part, the element-matching network must match this impedance. Furthermore, each matching network must be supplied with the proper input signal, in magnitude and phase, to ensure a match with the specified currents. If the active impedance has a negative real part, the matching network must be replaced with negative impedance. To avoid excess loss, this impedance can be replaced with a lossless two port, presenting this impedance to the antenna with its output signal then combined (added) to the overall input signal driving the array.

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CONTENTS

Figures (Contd.)

INTRODUCTION

Electrically small antennas are sometimes required by missile systems because of limited space, reduction in radar cross section, or desired operation at a longer wavelength.

Where a longer operating wavelength supplies the motivation, super-gain/super-directive arrays represent a potential solution. Antenna gain is defined as 4 π times the radiation intensity in a given direction divided by the net power accepted by the antenna (Reference 1). Radiation intensity is defined as the real part of the complex Poynting vector times r^2, where r is the distance from the antenna to the observation point. (This multiplication removes the 1/r dependence of the radiated electromagnetic fields.)

Therefore, antenna gain is closely related to the antenna efficiency that we define as the ratio of the total radiated power to this same input power. The efficiency is 1/4 π times the integral of the gain over the increment of solid angle on a sphere enclosing the antenna.

The requirement for high efficiency leads to the choice of a high-temperature superconductor (HTS) because antennas that are small with respect to a wavelength have a high-conductor loss as compared to their radiation resistance when conventional conductors are used.

Directivity is defined as the maximum directive gain. In turn, directive gain is the ratio of the radiation intensity in a certain direction to the average radiation intensity. The average radiation intensity is the total power radiated divided by 4 π, which is the average intensity per unit solid angle (steradian).

The directive gain is 4 π times the radiation intensity in a given direction divided by the power actually radiated. Antenna gain is the antenna efficiency times the directive gain, where both have the same reference direction, so that the maximum antenna gain is the efficiency times the directivity (Reference 1).

In general, the smaller the antenna or an antenna array is with respect to a wavelength, the greater the beamwidth and the lower the directivity is expected to be. Typically for an antenna array occupying some area, the directivity can be expected, as this area is made smaller, to decrease as four π times this area divided by the wavelength squared. This is the directivity of a uniformly excited rectangular aperture (Reference 1).

A superdirective array, then, is a small array that exhibits a much higher directivity than a uniformly excited rectangular aperture of the same area. A super-gain array is one that not only has a high directivity but shows much higher efficiency than might be expected, considering the usual ohmic (conductor) losses.

Although it seems counterintuitive, small arrays could, theoretically, have directivities exceeding this nominal value by properly driving the element currents in magnitude and

phase. Proof can be seen by considering two small dipoles separated by a fraction of a wavelength. The dipoles produce fields each according to the currents driving them, and the total field at any point in space is the superposition of these fields. It is important to note that the total power radiated is not the sum of the powers these two dipoles would radiate if each were considered in isolation.

For example, if the two antennas were separated by many wavelengths with currents in phase and equal in magnitude, the total power radiated would be essentially $2P_r$, where P_r is the radiated power produced by one dipole driven with this current. As the separation is decreased, the total power radiated would increase until, at zero separation, it becomes $4P_r$—the equivalent of driving one dipole with twice the current. Consequently, the directivity of the widely separated dipoles decreases to that of a single infinitesimal dipole—1.27.

On the other hand, if these dipoles were driven by equal currents 180 degrees out of phase, the two currents would still be $2P_r$ when the elements are widely separated. However, as the spacing is decreased, the power radiated decreases, going to zero when the separation is zero. Zero separation is equivalent to driving one dipole with zero current.

The directivity is a different matter. Imagine these dipoles as oriented in the Z-direction, spaced along the X-axis at L/2 and -L/2 about the origin. No radiation will occur in the YZ plane (X = 0) because the superposed fields will exactly cancel at that point. However, along the X-axis the superposed fields vary as $1 - \exp(-jkL)$.

If L is small then the total (superposed) field is small, but as long as L is greater than zero, some radiation will occur in the X-direction. For example, if $L = \lambda/16$, $1 - \exp(-jkL) = 0.0761 - j\ 0.3827$ with a magnitude of 0.3902. While this is not the $1 + \exp(jkL) = 1.9299 - j\ 0.3827$ with magnitude 1.9675, which would be the case if the currents were in phase, some power radiates in the X-direction but not in the Y-direction, and the directivity of the out-of-phase dipoles is greater than that of the in-phase dipoles.

This dependence of the directivity and total power radiated on the relative phases and magnitudes of the driving currents can be explained by antenna coupling. What is described is the inherent coupling between two dipoles or antennas that obviously become large as the antenna separation goes to zero. Additional coupling exists in the case of actual rather than theoretical antennas because of the presence of driving and mounting structures, ground planes, etc.

The example of two dipoles illustrates another unfortunate tendency of small superdirective arrays, the trade-off in power radiated for directivity. In going from something approaching $4P_r$ with in-phase driving currents to something just greater than zero power radiated with out-of-phase currents of the same magnitude, the radiation resistance of each dipole evidently is decreased because the power radiated is essentially the current magnitude squared times the radiation resistance.

Clearly, at least for conventional antenna designs (Reference 2), the conductor losses as compared to the radiation resistances are even more important for superdirective arrays than for the small antennas that form the elements of the array. This loss increases the need for HTS for both the elements and the matching circuits for the elements, and further increases the difficulty of designing matching circuits because the "Q" is increased.

This report examines the problem of matching into the active impedances, which are the result of this interelement coupling, and lays out a procedure for determining the required scattering parameters for the element-matching circuits given the driving currents.

THEORY

The radiation integral for the vector potential in the time harmonic case is

$$\bar{A} = \frac{\mu}{4\pi}\iiint \frac{\bar{J}\left(\bar{r'}\right)}{\left|\bar{r}-\bar{r'}\right|} e^{-jk\left|\bar{r}-\bar{r'}\right|} dv' \tag{1}$$

where

$\bar{J}$ = current density

For wire antennas—dipoles, monopoles, loops—this reduces to the integral

$$\bar{A} = \frac{\mu}{4\pi}\int \frac{\bar{I}\left(\bar{r'}\right)}{\left|\bar{r}-\bar{r'}\right|} e^{-jk\left|\bar{r}-\bar{r'}\right|} d\ell \tag{2}$$

where

$\bar{I}$ = current in the wire elements

The pattern and polarization, the magnitudes and phases of the electromagnetic fields, near and far, are determined by the vector potential $\bar{A}$. This relationship is true whether our antenna is a single continuous antenna or an array of individual antenna elements, each with a different magnitude and phase of the driving current.

Put another way, given an array of antennas, we can, in principle, shape the antenna pattern and determine the radiation direction by specifying the drive currents on the antenna elements without regard to how such drive currents could be realized in proper relative magnitude and phase in practice.

For commonly implemented antenna arrays where the spacing between elements is on the order of a half wavelength and the coupling between elements is small enough to be ignored, the problem of supplying the required currents can be solved in a straightforward manner. Conceptually, the input signal to the N-element array is divided by an N-way power divider into N signals of the proper magnitudes (Figure 1).

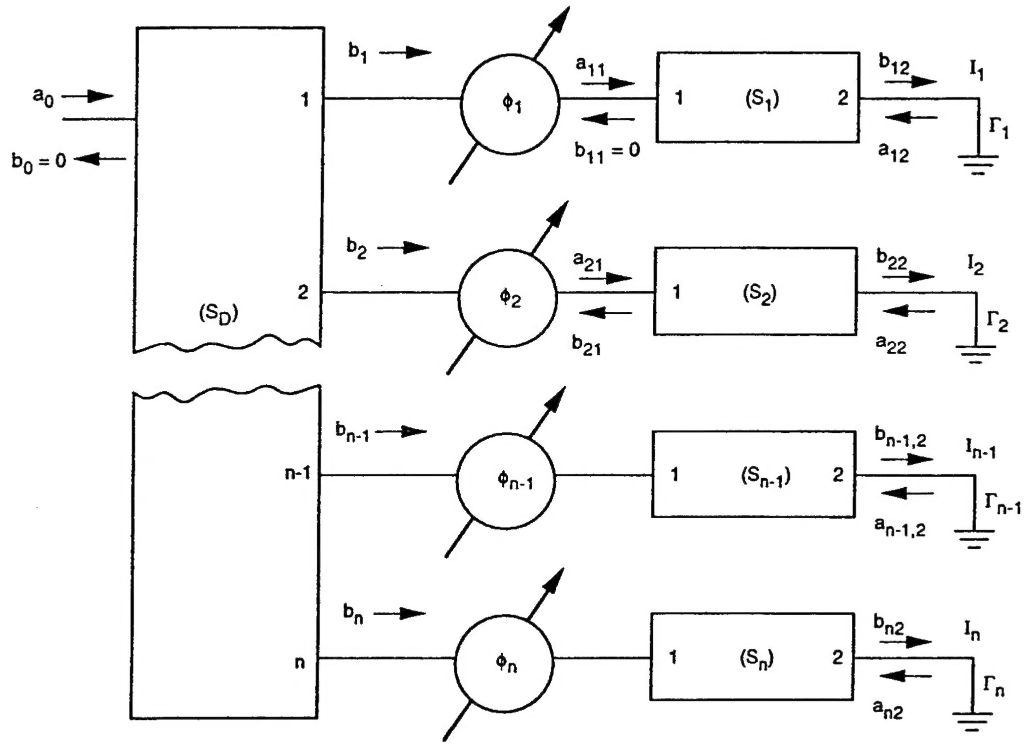

FIGURE 1. Power Divider, Phase Shifters, and Matching Circuits to Produce Specified Currents $I_1, I_2, ..., I_n$ on an Antenna Array Where Antenna Elements Are Represented by Their Reflection Coefficients, $\Gamma_1, \Gamma_2, ..., \Gamma_n$.

The power divider is designed to give zero reflection at the input port, if output ports 1 through N are terminated in the characteristic impedance Z_0. The (uneven) power divider is designed to give outputs b_1 through b_N of the proper magnitude to maintain the specified current magnitudes I_1 through I_N of the inputs to the antenna elements.

Each power divider output port is followed by an (ideal) phase shifter with the phase adjusted to maintain the proper phase relations (at a given frequency) between the currents $I_{1,2\text{-}N}$. These phase shifters are followed by matching networks (S_j), which match into the antenna element impedances such that $b_{j1} = 0$ at the matching network inputs.

If, as is often the case for the usual antenna array, the elements are all identical, $\Gamma_1 = \Gamma_2 = ... = \Gamma_n = \Gamma$, the problem is greatly simplified. In general,

$$Z_0 I_j = b_{j2} - a_{j2} = b_{j2}\left(1 - \Gamma_j\right) \tag{3}$$

For a lossless perfect match, the matching condition is

$$S_{j22} = \Gamma_j^* \tag{4}$$

where * indicates complex conjugate. Furthermore, in general,

$$b_{j2} = \frac{S_{j12} e^{j\phi_j} b_j}{1 - S_{j22}\Gamma_j} \tag{5a}$$

If the array elements are identical, making the matching networks identical (as, indeed, they must be, theoretically, to within an arbitrary phase shift) is convenient and Equation (5a) becomes

$$b_{j2} = \frac{S_{12} e^{j\phi_j} b_j}{1 - S_{22}\Gamma} \tag{5b}$$

It follows that

$$\frac{b_{j2}}{b_{k2}} = \frac{b_j e^{j\phi_j}}{b_k e^{j\phi_k}} = \frac{I_j}{I_k} \tag{6}$$

Therefore, the power divider itself needs only to produce outputs in which the ratios of its output magnitudes are the same as the relative magnitudes of the antenna currents, while the phase shifters must be adjusted to yield a relative phase shift, ϕ_j - ϕ_k equal to the relative antenna current phase shift, ϕ_{Ij} - ϕ_{Ik}.

The problem of nonidentical antenna elements is more involved but still straightforward. We have

$$\frac{I_j}{I_k} = \frac{b_{j2}\left(1 - \Gamma_j\right)}{b_{k2}\left(1 - \Gamma_k\right)} = \frac{S_{j12} e^{j\phi_j}\left(1 - \Gamma_j\right)}{1 - S_{j22}\Gamma_j} \cdot \frac{1 - S_{k22}\Gamma_k}{S_{k12} e^{j\phi_k}\left(1 - \Gamma_k\right)} \frac{b_j}{b_k}$$

or

$$\frac{b_j}{b_k} = \frac{S_{k12}\left(1-\Gamma_k\right)\left(1-S_{j22}\Gamma j\right)}{S_{j12}\left(1-\Gamma_j\right)\left(1-S_{k22}\Gamma_k\right)} \cdot \frac{e^{j\phi_k}}{e^{j\phi_j}} \frac{I_j}{I_k} \tag{7}$$

While Equation (7) is more complicated than Equation (6), all the values are known so finding the power divider ratios and the setting of the phase shifters is possible.

In our case, however, we are dealing with arrays and array elements that, although small with respect to a wavelength, are generally closely spaced with values of the antenna input currents, which differ widely in both phase and magnitude. We cannot ignore the interelement coupling because changing the current on any one antenna changes all the input currents, and this effect must be accounted for.

The procedure for active impedance matching in the case of nonnegligible interelement coupling is also, with pitfalls, straightforward, provided the array impedance (Z), admittance (Y), or scattering (S) matrix is known. Typically, this information is supplied by a design (CAD) program but could also be supplied by measurement.

The (S), (Y), and (Z) matrices are related by the matrix equations

$$(Z) = (Y)^{-1} = Z_0\left((U)+(S)\right)\left((U)-(S)\right)^{-1} \tag{8a}$$

and

$$(S) = \left((Z_0)+(Z)\right)^{-1}\left((Z)-(Z_0)\right) \tag{8b}$$

where

(U) = unit matrix
$(Z_0) = Z_0(U)$.

The assumption is made that the same reference impedance, Z_0, is used throughout.

The active impedance-matching procedure is easiest to deal with by considering the two-element array shown in Figure 2. If N-antennas are in the array, the array can be considered an N-port for purposes of active impedance matching.

Referring to the two-port network in Figure 2, the assumption is made that the currents I_1 and I_2 are specified to give a particular array performance—pattern, radiation direction—and that (Z) is known (or derivable from a known (Y) or (S) matrix).

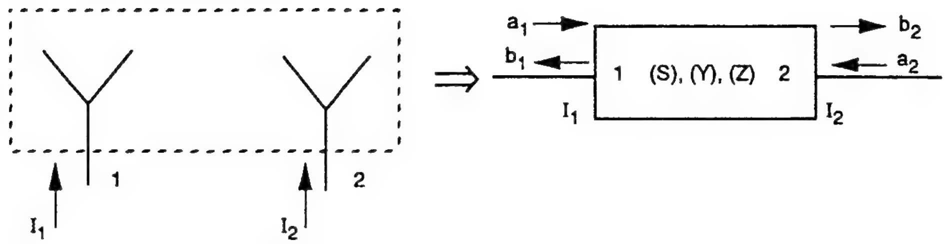

FIGURE 2. Two-Element (Antenna) Array as a General Two-Port Network With Attendant (S), (Y), and (Z) Matrices.

In this case

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \tag{9a}$$

$$V_2 = Z_{12}I_1 + Z_{22}I_2 \tag{9b}$$

and the active input impedances of ports 1 and 2 are then

$$Z_1 = V_1/I_1 = Z_{11} + Z_{12}\, I_2/I_1 \tag{10a}$$

$$Z_2 = V_2/I_2 = Z_{22} + Z_{12}\, I_1/I_2 \tag{10b}$$

It follows that $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$ are actual fixed, complex impedances, if we are to use a particular antenna array with specified antenna/input currents. The actual problem to be solved is then represented by Figure 3, where

$$\Gamma_i = \frac{Z_i - Z_0}{Z_i + Z_0}\,; i = 1,2 \tag{11}$$

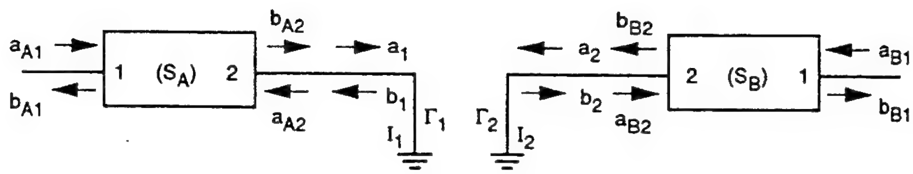

FIGURE 3. Two-Element Array as Represented by the Active Reflection Coefficients Γ_1, Γ_2, With Matching Networks (S_A) and (S_B).

In essence, we have decoupled the input ports of our antenna array. In so doing, since

$$I_1 = \frac{1}{Z_0}\left(b_{a2} - a_{a2}\right) \tag{12a}$$

$$I_2 = \frac{1}{Z_0}\left(b_{B2} - a_{B2}\right) \tag{12b}$$

designing matching networks for Γ_1 and Γ_2 is not sufficient. We must employ the proper drive levels a_{A1} and b_{B1} in magnitude and phase so that the specified currents, I_1 and I_2, are maintained. Failure to do so means not only are the currents at the antenna inputs not the designed ones, but the active impedances are changed and the matching networks no longer supply a match.

The extension to an N-element array is straightforward. Here the active input impedance at the j^{th} port is given by

$$Z_j = Z_{j1}\frac{I_1}{I_j} + Z_{j2}\frac{I_2}{I_j} + \ldots + Z_{jj} + \ldots + Z_{jN}\frac{I_N}{I_j} \tag{13}$$

The problem becomes one of finding the proper matching network for each Z_j and finding the correct magnitude and phase of the drive level for each matching network.

Returning to Figures 2 and 3—the simple example of a two-port network—rewrite Equations (10(a)) and (10(b)) as (Reference 2)

$$Z_1 = R_1 + jX_1 = R_{11} + \frac{|I_2|}{|I_1|} R_{12} \cos\phi - \frac{|I_2|}{|I_1|} X_{12} \sin\phi$$
$$+ j\left(X_{11} + \frac{|I_2|}{|I_1|} X_{12} \cos\phi + \frac{|I_2|}{|I_1|} R_{12} \sin\phi \right) \tag{14a}$$

$$Z_2 = R_2 + jX_2 = R_{22} + \frac{|I_1|}{|I_2|} R_{12} \cos\phi + \frac{|I_1|}{|I_2|} X_{12} \sin\phi$$
$$+ j\left(X_{22} - \frac{|I_1|}{|I_2|} R_{12} \sin\phi + \frac{|I_1|}{|I_2|} X_{12} \cos\phi \right) \tag{14b}$$

where ϕ is the relative phase shift between I_2 and I_1.

Depending on the angle ϕ, the relative magnitudes of I_1 and I_2, the magnitude of R_{12}, and the magnitude and sign of X_{12}, the active impedance, Z_1 or Z_2, may possess a negative resistance (real part), even though R_{11} and R_{22} must be positive.

Physically, we associate a negative resistance with amplification, but, in this case, the voltage wave leaving an antenna input port is larger in magnitude than the wave entering the port. The excess signal is supplied via coupling to the other ports. In Figure 2, if R_1 is negative, $|b_1| > |a_1|$ and the extra magnitude for b_1 is supplied by the input, a_2, to port 2 via the coupling between the antenna elements.

For the two-port network in Figure 2, only one port can exhibit a negative resistance for the active impedance, the other must be positive. Similarly, conservation of energy requires, for an N-element array, that at least one of the ports must show a positive active input resistance.

If the real part of Z_i is negative, substitution into Equation (11) always leads to $|\Gamma_i| > 1$. Conversely, if the real part is positive, $|\Gamma_i| < 1$, and if it is zero, $|\Gamma_i| = 1$.

The process of active impedance matching divides naturally into two cases: where $R_i > 0$ and $R_i \leq 0$. We will deal with the simplest case first, where the real part of the active impedance is positive, as determined by Equations (14(a)) and (14(b)).

First, the active reflection coefficients are found from Equation (11). A perfect match implies $b_{A1} = b_{B1} = 0$ (Figure 3). To avoid loss in the matching network, (S_A) and (S_B)

must be unitary, leading to the following lossless two-port network conditions (Reference 3):

$$|S_{H11}| = |S_{H22}| \tag{15a}$$

$$|S_{H12}|^2 + |S_{H22}|^2 = 1 \tag{15b}$$

$$2\phi_{H12} = \phi_{H11} + \phi_{H22} \pm \pi \,;\, H = A,B \tag{15c}$$

(The assumption is made that the S_{ij} are voltage wave-scattering coefficients and the same reference impedance, Z_0, is used everywhere).

To match any reflection coefficient, $\Gamma(|\Gamma| < 1)$, with a <u>lossless</u> two-port network, we must have

$$S_{H22} = \Gamma_i^* \quad ; \quad H = A,B \quad , \quad i = 1,2 \tag{16}$$

Thus, knowing Γ_i, we know $|S_{H22}|$, ϕ_{H22} and, with Equations (15(a)) through (15(c)), $|S_{H11}|$ and $|S_{H12}|$. One of the phase shifts in Equation (15(c)), ϕ_{H12} or ϕ_{H11}, can be chosen arbitrarily. This is readily seen by a Gedanken experiment: adding a length of line of characteristic impedance Z_0 and phase shift ϕ_L to port 1 creates a new two-port network with reflection coefficients of the same magnitude, with ϕ_{H12} increased by ϕ_L and ϕ_{H11} by $2\phi_L$.

Although the choice of ϕ_{H11} or ϕ_{H12} is arbitrary, some choices can lead to difficulty. For example, choosing $\phi_{H11} = 0$ may lead to a scattering matrix (S) that cannot be converted to a (Z) matrix since

$$\mathrm{DET}|(U) - (S)| = 0 \tag{17}$$

Because working with (Z) rather than (S) may be desirable at UHF/VHF frequencies, where small lumped elements are viable, it is probably wise to avoid the situation of Equation (17). A choice that works well is

$$\phi_{H11} = \phi_{H22} \quad ; \quad H = A,B \tag{18}$$

Having made this choice, we have from Equations (15(a)) through (15(c))

$$|S_{H11}| = |S_{H22}| = |\Gamma_i| \quad ; \quad H = A, i = 1 \; or \; H = B, i = 2 \tag{19a}$$

$$|S_{H12}|^2 + |\Gamma_i|^2 = 1 \tag{19b}$$

$$2\phi_{H12} = -2\phi_i \pm \pi \tag{19c}$$

where

$\Gamma_i = |\Gamma_i| \exp(j\phi_i)$

Although the scattering parameters of the matching networks have now been specified, the proper input currents I_1 and I_2 must be maintained, if there is to be no reflection loss ($b_{A1} = b_{B1} = 0$ in Figure 3). Thus, the drive levels, a_{A1} and a_{B1}, must be found as a function of I_1 and I_2 for the array to work as designed.

From Figure 3: $b_{A2} = S_{A12}\, a_{A1} + S_{A22}\, \Gamma_1\, b_{A2}$ or $b_{A2} = \dfrac{S_{A12} \cdot a_{A1}}{1 - |\Gamma_1|^2}$. Then, because $a_{A2} = \Gamma_1 b_{A2}$, we have

$$I_1 = \frac{1}{Z_0}(b_{A2} - a_{A2}) = \frac{1}{Z_0}(1 - \Gamma_1) b_{A2} = \frac{1}{Z_0}(1 - \Gamma_1) \frac{S_{A12} a_{A1}}{1 - |\Gamma_1|^2}$$

and thus

$$a_{A1} = \frac{Z_0 \left(1 - |\Gamma_1|^2\right) I_1}{S_{A12}(1 - \Gamma_1)} \tag{20}$$

By analogy

$$a_{B1} = \frac{Z_0 \left(1 - |\Gamma_2|^2\right) I_2}{S_{B12}(1 - \Gamma_2)} \tag{21}$$

Equations (20) and (21) are special cases of the general equation

$$a_1 = \frac{Z_0(1 - S_{22}\Gamma)}{S_{21}(1-\Gamma)} I \tag{22}$$

which relates the voltage wave, a_1, of the input of port 1 of a two-port network to the current, I, through the termination at port 2. This equation applies regardless of match or loss in the two-port network, reducing to Equations (20) and (21) for the lossless matched cases.

At this point the active impedance matching is completed, although the problem remains of designing a power divider to supply a_{A1} and a_{B1}, as they are assumed coherent, and thus proceeding ultimately from the same source or generator. The complete block diagram for a two-element array is shown in Figure 4.

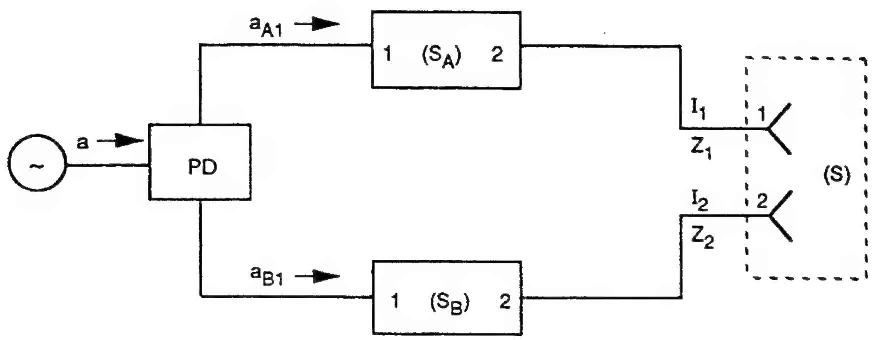

FIGURE 4. Two-Element Array Matched System in Which Neither Input Impedance Z_1 nor Z_2 Has a Negative Real Part. PD is a power divider designed to yield A_{A1} and A_{B1} in the proper magnitudes and phases to give the design values of the input currents I_1 and I_2.

For the case where all the active input impedances, Z_i, have a positive real part, the extension to an N-element array (N-port) is obvious and straightforward.

We now deal with the case for negative real parts of the active input impedances, Z_i, again using Figure 2 as an example. Assume that Z_1 exhibits a negative resistance; then Z_2 must have a positive real part. A matching network for port 2 can be formulated as previously by setting $S_{B22} = \Gamma_2^*$.

However, matching port 1 with a passive network is impossible because $|\Gamma_1| \geq 1$. (The equal sign is employed because, strictly speaking, if there is zero loss, $R_1 = 0$, and you cannot match into Z_1 because no physical means of dissipating power exists.) That $|\Gamma_1| \geq 1$ can easily be shown.

$$|\Gamma_1|^2 = \left|\frac{Z_1 - Z_0}{Z_1 + Z_0}\right|^2 = \left|\frac{-|R_1| + jX_1 - Z_0}{-|R_1| + jX_1 + Z_0}\right|^2$$

or

$$|\Gamma_1|^2 = \frac{-(|R_1| + Z_0) + jX_1}{(Z_0 - |R_1|) + jX_1} \cdot \frac{-(|R_1| + Z_0) - jX_1}{(Z_0 - |R_1|) - jX_1}$$

and

$$|\Gamma_1|^2 = \frac{(|R_1| + Z_0)^2 + X_1^2}{(Z_0 - |R_1|)^2 + X_1^2} \tag{23}$$

Since $(Z_0 + |R_1|)^2 \geq (Z_0 - |R_1|)^2$, we must have $|\Gamma_1| \geq 1$.

However, port 1 must be terminated in a reflection coefficient that produces the proper current, I_2, at this port. Thus, we need to deal with the situation depicted in Figure 5.

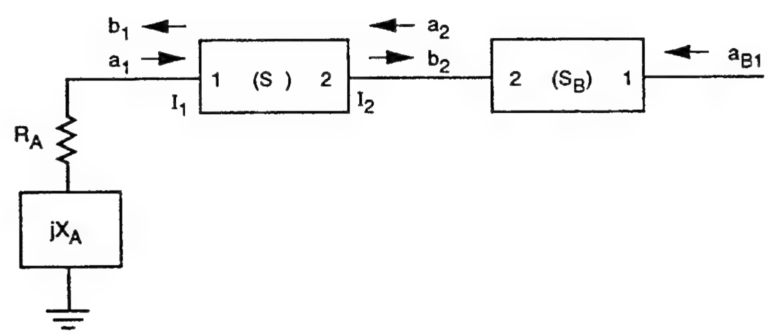

FIGURE 5. Termination of Port 1 of a Two-Element Array in an Impedance Z_a to Maintain Required Current, I_1, at That Port for the Case Where the Active Impedance Z_1 Has a Negative Real Part.

The first step is to determine the value of Z_A needed to ensure the correct current, I_1, of port 1 of the array. Because Figure 5 can be represented by Figure 6, this determination is straightforward. We find that

$$a_1 = \Gamma_A b_1 = \Gamma_A \Gamma_1 a_1 \tag{24}$$

Therefore

$$\Gamma_A = 1/\Gamma_1 \tag{25}$$

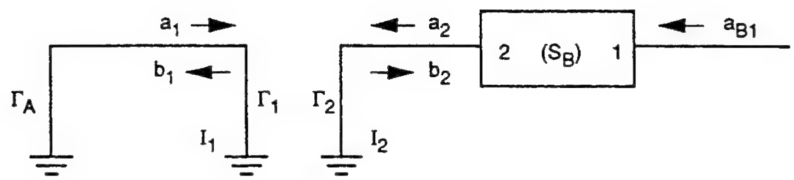

FIGURE 6. Represents Figure 5 by Breaking Figure Into Two Parts Using the Concept at Active Impedance and Reflection Coefficients.

The real part of Z_1 is negative, therefore

$$Z_1 = -|R_1| + jX_1 \tag{26}$$

From Equation (23) it follows that

$$\frac{Z_A - Z_0}{Z_A + Z_0} = \frac{Z_1 + Z_0}{Z_1 - Z_0}$$

or

$$Z_A = -Z_1 \tag{27a}$$

$$R_A = |R_1| \tag{27b}$$

$$X_A = -X_1 \tag{27c}$$

The required input drive level, a_{B1}, for a specific value of the current I_2 and, hence, I_1, can still be found from Equation (21). However, since the ratio I_2/I_1 generally determines the antenna array performance in terms of pattern, sidelobe level, and directivity, this ratio evidently will remain unchanged regardless of a_{B1}, once (S_B) and Z_A are determined for Figure 5.

If Z_2 instead of Z_1 has the negative real part, we obviously would design a matching network (S_a) as before and terminate port 2 of the array with $Z_B = |R_2| - jX_2$. Furthermore, the extension to an N-element array also is obvious.

Unfortunately, a serious problem exists in dealing with an active negative resistance in the fashion of Figures 5 and 6. Because R_A is a positive real resistance, it represents actual power loss, lowering the radiation efficiency of an array. This efficiency loss tends to negate the reason for using HTS antenna elements in the first place.

One solution to this problem is shown in Figure 7. Port 1 feeds a lossless two-port network (S_A) with

$$S_{A22} = \frac{1}{\Gamma_1} \tag{28}$$

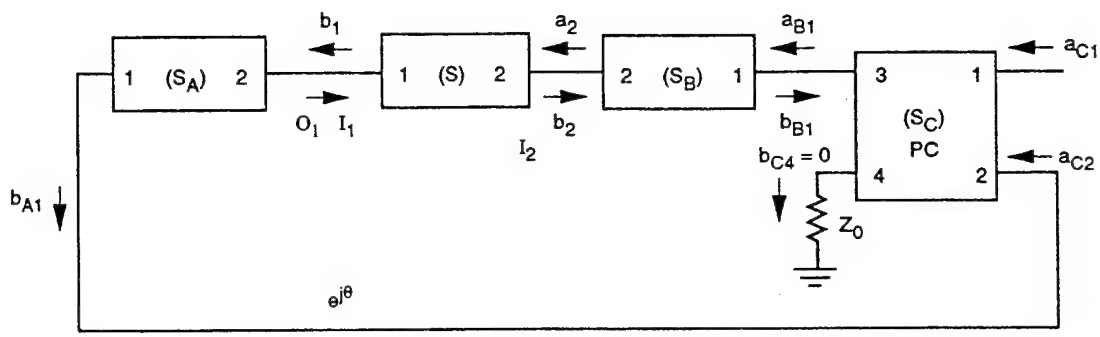

FIGURE 7. Lossless Two-Port Network, (S_A), to Feed Back the Excess Signal, (b_1) > (a_1), in the Case of an Active Negative Resistance of Port 1 of the Array.

and the excess signal, b_{A1}, is fed to a reflectionless power combiner, (S_c), to be combined with the overall input signal, a_{c1}, to produce a_{B1}. This combiner, to work properly, requires $S_{c11} = S_{c22} = 0$ and the coupling coefficients, S_{c14} and S_{c24}, must be chosen such that

$$b_{c4} = S_{c14}a_{c1} + S_{c24}a_{c2} = 0 \tag{29}$$

Because S_{A22} is fixed by Equation (28), $|S_{A11}|$ and $|S_{A12}|$ are also known via Equations (15(a)) and (15(b)). For a reflectionless combiner, S_{A11} evidently has no effect on the operation of the circuit. There is no need to obtain a value for ϕ_{A11}, nor for that matter, ϕ_{A12}, except that as

$$a_{c2} = b_{A1}e^{j\theta} = |S_{A12}|b_1 e^{j\theta} e^{j\phi_{A12}} = |S_{A12}||b_1|\exp(\theta + \phi_{A12} + \phi_{b1}) \tag{30}$$

the phase shift, θ, must be such that Equation (29) is satisfied. Because b_{B1} and a_{c4} are zero in Figure 7, the values of S_{c33} and S_{c44} are not required in the analysis. But as a practical matter, to take care of any residual reflections from S_{B11} and Z_0 due to manufacturing errors and to simplify the mathematics, we set these values equal to zero also.

It is also evident from the circuit as shown that $S_{c12} = 0$ is required to avoid dissipating power in the source impedance. By the previous arguments, we will set $S_{c34} = 0$ as well. The combiner scattering matrix now has the form

$$(S_c) = \begin{pmatrix} 0 & 0 & S_{c13} & S_{c14} \\ 0 & 0 & S_{c23} & S_{c24} \\ S_{c13} & S_{c23} & 0 & 0 \\ S_{c14} & S_{c24} & 0 & 0 \end{pmatrix} \tag{31}$$

To make (S_c) lossless, the unitary matrix condition is imposed, leading to the following set of equations:

$$|S_{c13}|^2 + |S_{c14}|^2 = 1 \tag{32a}$$

$$S_{c13}S_{c23}^* + S_{14}S_{c24}^* = 0 \tag{32b}$$

$$|S_{c23}|^2 + |S_{c24}|^2 = 1 \tag{32c}$$

$$|S_{c13}|^2 + |S_{c23}|^2 = 1 \tag{32d}$$

$$S_{c13}S_{c14}^* + S_{23}S_{c24}^* = 0 \tag{32e}$$

$$|S_{c14}|^2 + |S_{c24}|^2 = 1 \tag{32f}$$

From Equations (32(a)) and (32(d)), it is seen that

$$|S_{c14}| = |S_{c23}| \tag{33a}$$

and then from Equations (30(a)) and (30(f))

$$|S_{c13}| = |S_{c24}| \tag{33b}$$

From Equations (32(b)) and (32(c)), we have

$$\phi_{c13} - \phi_{c23} = \phi_{c14} - \phi_{c24} \pm \pi \tag{34a}$$

$$\phi_{c13} - \phi_{c14} = \phi_{c23} - \phi_{c24} \pm \pi \tag{34b}$$

and these are seen to be the same equation. As far as the phases are concerned, one equation has four unknowns, so three of the phases are arbitrary. For symmetry and mathematical simplicity, we choose $\phi_{c13} = \phi_{c24} = 0$ and $\phi_{c23} = \phi_{c14}$. It follows that $\phi_{c14} = \pm\pi/2$. Choosing the minus sign, Equation (31) becomes

$$(S_c) = \begin{bmatrix} 0 & 0 & S_{c13} & S_{c14} \\ 0 & 0 & S_{c14} & S_{c13} \\ S_{c13} & S_{c14} & 0 & 0 \\ S_{c14} & S_{c13} & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & S_{c13} & j\sqrt{1-S_{c13}^2} \\ 0 & 0 & j\sqrt{1-S_{c13}^2} & S_{c13} \\ S_{c13} & j\sqrt{1-S_{c13}^2} & 0 & 0 \\ j\sqrt{1-S_{c13}^2} & S_{c13} & 0 & 0 \end{bmatrix} \tag{35}$$

Equation (35) is the basic equation for the scattering matrix of a dual-directional coupler.

To recapitulate, we have, in principle, found the scattering parameter values for (S_B) and (S_A), and the values of (S), Z_1, Z_2, Γ_1, Γ_2, I_1, and I_2, or at least the ratio of I_2/I_1, are known. The values for (S_c), S_{c14}, and S_{c13}, as well as θ, remain to be determined.

The easiest way to find these values is to specify particular values for I_1 and I_2, even though only their ratio is important for array performance. For example, specify $I_1 = 1$ and I_2 is then equal to the given ratio. Once constructed properly, the circuit will maintain the proper ratio, I_2/I_1, as a_{c1} is varied. At the specified currents I_1 and I_2, Figure 7 can be replaced with Figure 8.

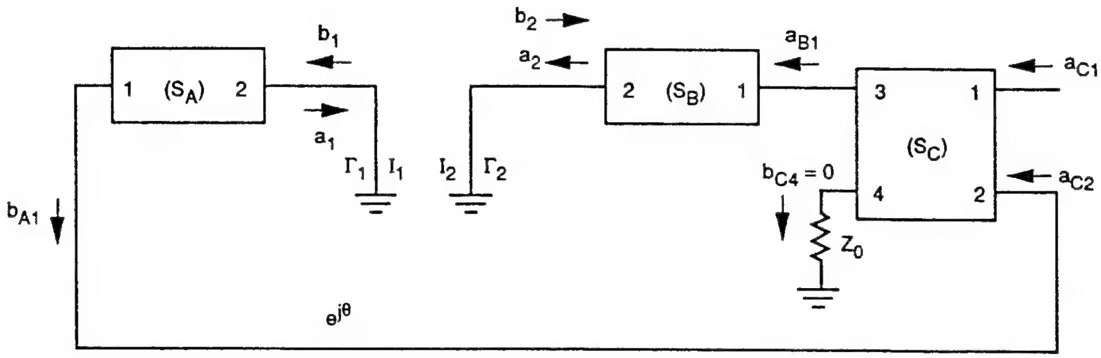

FIGURE 8. Circuit Equivalent to Figure 7 When the Proper Operating Currents I_1 and I_2 (or Their Ratio) Are Maintained.

The voltage wave b_{A1} can be found from (S_A) and the known I_1, at which time it will also be a known quantity, i.e.,

$$b_{A1} = \frac{S_{A12} Z_0 \Gamma_1}{1 - \Gamma_1} I_1 \tag{36}$$

The value of a_{B1} is also found from Equation (25) and added to the list of known quantities.

From Figure 8 we have the equations

$$a_{B1} = S_{c13} a_{c1} + S_{c14} e^{j\theta} b_{A1} \tag{37a}$$

$$0 = S_{c14} a_{c1} + S_{c13} e^{j\theta} b_{A1} \tag{37b}$$

Solving Equation (37(b)) for a_{c1} and substituting into Equation (37(a)), we have

$$a_{B1} = \frac{-S_{c13}^2}{S_{c14}} e^{j\theta} b_{A1} + S_{c14} e^{j\theta} b_{A1}$$

Substituting for S_{c14} from Equation (35), where S_{c13} is a real number,

$$\frac{a_{B1}}{b_{A1}} = \frac{\left(j\sqrt{1-S_{c13}^2}\right)^2 - S_{c13}^2}{j\sqrt{1-S_{c13}^2}} e^{j\theta} = \frac{-e^{j\theta}}{j\sqrt{1-S_{c13}^2}} \tag{38}$$

then

$$\frac{|a_{B1}|^2}{|b_{A1}|^2} = \frac{je^{-j\theta}}{\sqrt{1-S_{c13}^2}} \cdot \frac{-je^{j\theta}}{\sqrt{1-S_{c13}^2}} = \frac{1}{1-S_{c13}^2}$$

or

$$S_{c13}^2 = 1 - \frac{|b_{A1}|^2}{|a_{b1}|^2} \tag{39}$$

A glance at Figure 7 shows $|a_{B1}|^2 \geq |b_{A1}|^2$ by conservation of energy. The positive square root is used to find S_{c13}, as we have previously assumed $\phi_{c13} = 0$.

S_{c14} is defined in Equation (35), so the only remaining quantity is the phase shift, θ. Substituting Equation (39) into Equation (38) yields

$$\frac{a_{B1}}{b_{A1}} = \frac{je^{j\theta}}{|b_{A1}|/|a_{B1}|}$$

or

$$je^{j\theta} = \frac{|b_{A1}|}{|a_{B1}|}\frac{a_{B1}}{b_{A1}} = \frac{e^{j\phi_{aB1}}}{e^{j\phi_{bA1}}}$$

or

$$\theta = \phi_{aB1} - \phi_{bA1} - \frac{\pi}{2} \tag{40}$$

Although the mathematical arguments involved in the case of a negative real part to an active impedance at an array port have been more complicated, the actual circuit is no more complicated than the case where the active resistances are all real, as can be seen by comparing Figures 7 and 4. In Figure 4, we have a reflectionless power divider, PD,

whereas in Figure 7, we have a power combiner, PC. The phase shift $e^{j\theta}$ was shown extraneous to the combiner, PC, in Figure 7; but, in practice, could be included as part of the combiner design. In fact, an equivalent phase shift is needed for the power divider, PD, in Figure 4—by implication the phase shift is just included in the power divider.

The equivalence of Figures 4 and 7 becomes, perhaps, even clearer if you examine the two-port network, (S_A), in Figure 7 more closely. Even though (S_A) does not, strictly speaking, form a match to Z_1, it is a lossless match to some load.

Since $Z_1 = -|R_1| + jX_1$ and $S_{A22} = 1/\Gamma_1$, we see

$$S_{A22} = \frac{-|R_1| + jX_1 + Z_0}{-|R_1| + jX_1 - Z_0} = \frac{|R_1| - Z_0 - jX_1}{|R_1| + Z_0 - jX_1} \tag{41}$$

From Equation (16) we know that S_{A22} must match some load, Γ_ℓ, such that $\Gamma_\ell = S^*_{A22}$ or

$$\Gamma_\ell = \frac{|R_1| + jX_1 - Z_0}{|R_1| + jX_1 + Z_0} \tag{42}$$

That is, if we ignored the fact that $Z_1 = -|R_1| + jX_1$, treated it as if it were $Z_1 = |R_1| + jX_1$, and designed the appropriate matching network for this impedance, we would have the correct design for (S_A)! Figure 7 can be as well represented by Figure 4 as for the case of positive real parts for the active impedances, except for changing the letters PD to PC and deleting the wave a_{A1}, changing the direction of its arrow and replacing it with b_{A1}.

The case where Z_2 has the negative real part instead of Z_1 is obviously handled in the same fashion. For an N-element array where some of the elements have negative real parts and some positive, the approach to the matching networks is the same. Our power combiner has to be designed, if more than one element has a positive real part, as both a combiner and divider.

NUMERICAL EXAMPLES

This section contains numerical examples of active impedance-matching networks for a colocated magnetic loop and electric dipole array.

MOTIVATION

The motivation for such an investigation arose from the discovery that the active input resistances of such an array increased when the antenna elements were excited in

quadrature (Reference 2). Correspondingly, the Q associated with such an array was significantly lower than the Q of isolated elements in free space. With lower Q comes the potential ultimate payoff of increased bandwidth for electrically small antennas.

Wheeler (Reference 4), Chu (Reference 5), and others have formulated relationships governing the efficiency and bandwidth versus the electrical size of single-mode (i.e., electrical or magnetic) antennas. However, in these developments the realtionships derived between bandwidth and efficiency apparently did not address the simultaneous presence of both types of antennas. Therefore, the following heuristic argument for mixed-mode antenna arrays is postulated. First, consider the energy flow in single magnetic loop (inductive and its associated matching network (capacitive)). The energy oscillates back and forth between storage in the magnetic field of the antenna and the electric field in the matching network capacitor. In the steady state the antenna radiates a small amount of energy during the portion of the cycle when the magnetic field is high (i.e., high current). Correspondingly, a small amount of energy is supplied from the external generator. For a single electric dipole, a dual model can be postulated.

For a mixed-mode array, we can postulate that energy is radiated during both portions of the cycle: from the loop during the period of high-magnetic field and from the dipole during the period of high-electric field. In this way the radiation is increased beyond that of the isolated single elements. This heuristic argument was initially used to account for the apparent increase in the active resistance of the mixed-mode array.

MUTUAL COUPLING EFFECTS

Upon closer examination, the active resistances of a quadrature-fed mixed-mode array were observed to be opposite in sign. One of the elements exhibited a negative resistance. In circuit theory a negative reisistance is usually associated with power generation. In this case, the negative resistance indicated that power was flowing from the element rather than being supplied to it. Thus, the following physical picture is suggested. For a quadrature-fed, colocated, mixed-mode array, a very strong mutual coupling exists between elements. The increase in resistance levels is due, in part, to power flow, which is mutually coupling to the adjacent element. Power entering one element is not only radiated but is coupled to the load connected to the adjacent element. Thus, while the bandwidth increase due to the lower Q (higher resistance levels) was welcomed, the lower efficiency due to the power loss in the load of the adjacent antenna element was not. Wheeler's condition governing the efficiency bandwidth product seems to be reasserting itself.

FEEDBACK SOLUTION PROPOSED

The solution delineated in the Introduction and Theory sections of this report was proposed to couple the power emanating from the adjacent element back to the input of the driven element in such a way as to maintain the proper quadrature current excitations and impedance matching. A quadrature coupler was envisoned to provide the required coupling characteristics. The Theory section of this report derives the required coupling coefficient and differential phase requirements as functions of the desired currents and resulting active reflection coefficients at the antenna ports. The approach seemed feasible, but further numerical examples were deemed necessary to determine the achievable bandwidth of such a feedback approach.

The work presented in this report is a collection of MathCad and Touchstone computer programs, which analyze the mixed-mode array with feedback matching.

ANTENNA ARRAY MODELLED USING NEC-3D

The antenna geometry (Figure 9) consisted of a center-fed dipole oriented along the z-axis with a length of 0.02 λ and wire radius of 0.001 λ. Surrounding the dipole (in the y-z plane) is a square loop with side length 0.025 λ and wire radius 0.001 λ. The loop is fed at the point x = 0, y = 0.0125 λ, z = 0 and the dipole is fed at the origin. The dipole is modelled using five segments and the loop is modelled using five sements per side. Lossless conductors are assumed. The extended kernal of NEC-3D was used.

The Y-parameters for the array were generated by sequentially exciting each port with 1 volt (with remaining ports short circuited) and calculating the current flow at each port. Appendix A contains a printout of the Y-matrix as a function of frequency for the antenna geometry described.

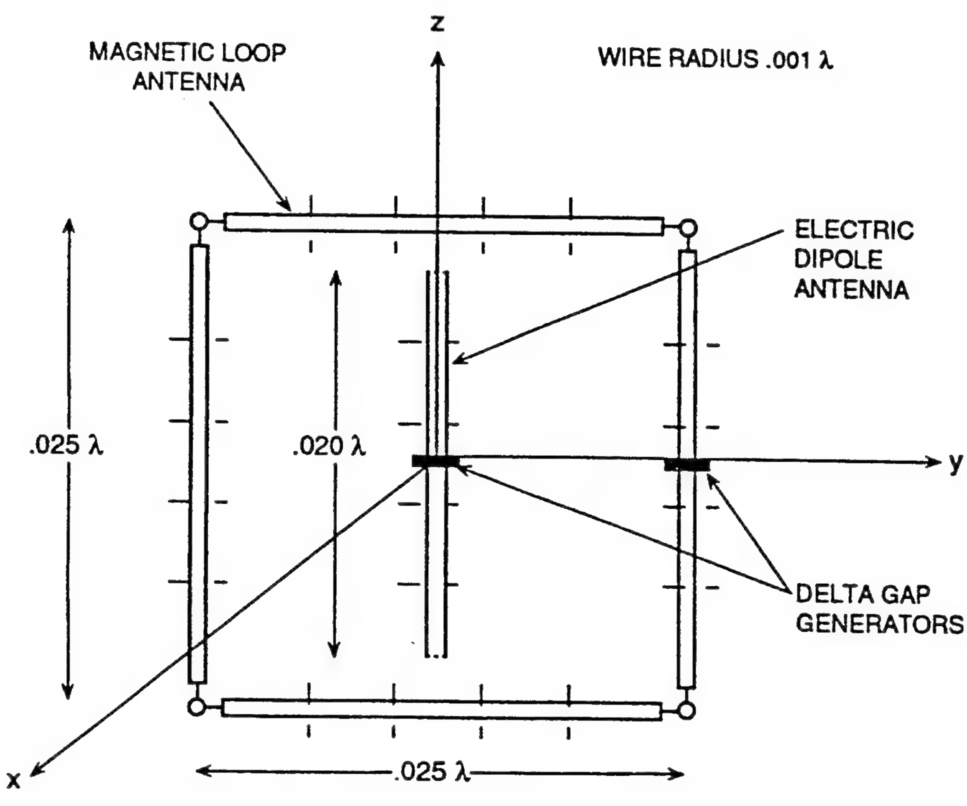

FIGURE 9. Colocated Antenna Geometry for Method of Moments Analysis.

We soon discovered that small nonsymmetries between Y12 and Y21 in the Y-matrix caused significant errors in computing the amplitudes of the waves entering and emanating from the matching networks of the mixed-mode array. To correct the problem reciprocity was enforced by using the averaged value for both Y12 and Y21. Appendix B contains a printout of the Y-matrix with symmetry enforced.

MATHCAD PROGRAM WRITTEN TO ANALYZE FEEDBACK MATCHING

The MathCad program in Appendix C was written to design the feedback-matching networks for the mixed-mode antenna. The design procedure is outlined as follows.

1. The Y-matrix (calculated by NEC) is read in as input data.

2. The Z-matrix is calculated by taking the inverse of the Y-matrix.

3. The required current amplitude and phases are calculated based on the requirement for zeroing the radial component of the reactive energy (Equations (40) and (41), Reference 2).

4. The active impedances of the array are calculated.

The MathCad program displays the active impedance results for both the dipole and loop. Notice that the dipole resistance is negative, indicating that power is being coupled from the loop to the dipole. The dipole reactance is quite high. Notice the Q values are much lower than would be observed for isolated elements (i.e., Q = 36 for dipole, 31 for loop versus Q >1000 for isolated elements).

5. The scattering parameters of a lossless matching network are derived (using equations derived in the the Theory section of this report), which match the active impedances to 50 ohms. Selection of 50 ohms as the transformed impedance level is arbitrary but commonly chosen. (Selecting a lower impedance value for bandwidth enhancement considerations may be desirable.) The matching networks are assumed lossless and symmetric. The method of S-parameter calculation differs, depending on whether the port exhibits a positive or negative active resistance.

6. The Z-matrices of the required matching networks are computed from the S-matrices.

7. A tee-network-matching topology was arbitrarily selected. The element values (capacitor and inductors) were calculated at each frequency. As shown in Appendix C, the required matching values are not constant with frequency. In the Touchstone simulations to follow, the midfrequency (500 MHz) values were selected. This simplification certainly limits the achievable bandwidth performance. Other circuit topologies that could reduce this variability and improve bandwidth should be investigated in the future.

8. The amplitude and phase of the waves incident upon the loop-matching network and emanating from the dipole-matching network are computed. The graph in Appendix C compares the wave amplitudes versus frequency. Notice that the amplitude of bA1 (the wave emanating from the dipole-matching network) is greater than the amplitude of aB1

(the wave incident upon the loop matching network), which violates the conservation of energy. Upon closer examination, the origin of this anomaly was traced to the nonreciprocal Y12 and Y21 values computed by NEC. Reciprocity can be enforced by using the average values of Y12 and Y21, as shown in Appendix B.

Appendix D contains the results when Y-matrix reciprocity is enforced. Notice in this case the amplitude of aB1(incident wave on loop) is greater than bA1 (emanating wave from dipole) and power is conserved. Furthermore, as will be demonstrated in the Touchstone simulations, if the power delivered to the load (on port 2 of the coupler) is forced to zero by proper selection of differential phase shift, the difference in power represented by amplitudes of aB1 and bA1 exactly equals the radiated power.

9. The required coupling parameter k and the amount of additional phase shift can be calculated. Appendix E is included to show the computation of additional phase shift required to provide cancellation at port 2 of an ideal quarter-wavelength directional coupler. The phase of the direct and coupled paths is compared at 500 MHz (Appendix E), illustrating the signal cancellation at port 2.

TOUCHSTONE ANALYSIS OF MIXED-MODE FEEDBACK MATCHING

Case I. Analysis Without Feedback

Apendixes F, G, and H contain Touchstone simulations for feedback matching of the mixed-mode antenna described.

Appendix F analyzes only the mixed-mode array and matching networks. Appendix F also contains the Touchstone circuit file, a diagram of the circuit file, and the Y-parameters of the mixed-mode antenna whose geometry has previously been described. Reciprocity has not been imposed for this case. Also displayed in Appendix F are the S-parameters (in a 50-ohm system) calculated for the array based on the NEC3D Y-parameters. Notice only a very slight nonreciprocity in the S12 and S21 values. Also included in Appendix F are tee-section matching networks for the loop and dipole and the scattering parameters for the combined antenna and matching networks. Notice the agreement between the phase angle of S21 and that shown in Appendixes C and D for the phase angle of b_{A1}/a_{B1}. Appendix further contains Touchstone's calculation of active impedances for the dipole and loop. Good agreement with MathCad results is observed (see Appendix C). Wave calculations for the case in which the loop-matching network is excited by a 50-ohm generator and the dipole-matching network is terminated in a 50-ohm load. As shown the bulk of the input power (95.3%) is simply coupled directly to the output load. Only 4.7% of the input power is actually radiated. A plot is included of the transmission coefficient as a function of frequency. The half-power (0.707) bandwidth is approximately 16 MHz for a Q of (500/16) or 31, which agrees with our earlier estimate of Q. The efficiency (in percent) bandwidth (in percent) product of the array (with no feedback) is $0.047 \times 0.032 = 1.47{*}10^{-3}$.

According to Wheeler the gain bandwidth product of an electrically small antenna is given by

$$Eff * BW = 8/3*\pi^3*(L/\lambda)^3$$

where L is the radius of a sphere that encloses the antenna.

Choosing the diagonal of the square loop ($D = 0.025\lambda*1.414 = 0.0353\lambda$) equal to the desired sphere diameter, a Wheeler efficiency bandwidth product of 4.55×10^{-4} is predicted. If this interpretation of the Wheeler condition is correct, an increase of a factor of 3 in the efficiency-bandwidth product is observed. Much futher study and empirical verification is needed, however, before definitively declaring an improvement over the Wheeler limit.

Appendix F contains the voltages and currents predicted by Touchstone for the mixed-mode antenna and tee-section matching with no feedback. The current ratio calculated by Touchstone is slightly different (+5%) from the prescribed current ratio calculated by the MathCad program in Appendixes C and D. The slight variation may well be due to the apparent nonreciprocity of the antenna Y matrix. Further study is needed to isolate the origin of the difference. Appendix F also contains a circuit configuration that allows Touchstone to automatically normalize calculated current values to the dipole current.

Case II. Analysis with Nonoptimum Feedback—Zero Additional Phase Shift, Weak Coupling

Appendix G contains the Touchstone circuit file, circuit diagram, and data. The incident and reflected waves are calculated at the coupler ports. Incident waves a_2 and a_4 are zero as expected; a_1 (the input) is defined as unity, and a_3 is observed to be greater than unity. These results point out an interesting aspect of feedback not previously appreciated. A significant amount of stored energy exists in the feedback loop. These results will be important in assessing the bandwidth performance of mixed-mode arrays with feedback matching. Appendix G includes the values calculated for the reflected waves from the coupler ports. Relected waves b_1 and b_3 are zero, indicating good impedance matching. Wave b_4 is greater than unity due to stored energy in the feedback loop. Wave a_3 is smaller than b_4 due to radiation from the antenna. Appendix G also contains a plot of the transmission to port 2 of the coupler. Significant differences are noted as compared to the previous case with no feedback (see Appendix F). The frequency response is nonsymmetric with very little cancellation observable. The bandwidth has increased slightly to 17.5 MHz.

Case III. Analysis With Optimum Feedback (Touchstone Optimizer)

Appendix H contains the results for the case of optimum feedback. The data in Appendix H were generated before the conditions of optimum feedback contained in Appendix E had been finalized. Optimum feedback was determined by using Touchstone's optimizer and simultaneously optimizing for minimum b_1 and b_2. Optimum coupling and differential phase shift values were very close to those predicted in Appendix E.

Parameter	Touchstone	Predicted
Coupling factor, k	0.99822	0.99788
Added differential phase shift, θ	63.44	63.41

Several significant differences were noted in the Case III simulation. As shown in Appendix H, the active impedance values observed were quite different from previous cases. The resistance of the dipole dropped by 12%. The resistance of the loop increased by 12%. Correspondingly, the excitation current ratio also shifted slightly.

Parameter	Optimum feedback	Weak feedback	No feedback
I Loop	8.946	0.598	0.567
I Dipole	1.936	0.112	0.106
Ratio (IL/ID)	4.621	5.339	5.349

Clearly, the current amplitudes have increased substantially (a factor of 15 to 18) under conditions of optimum feedback.

Appendix H also contains the calculated values for the incident and reflected waves at the input to the matching networks. Notice the slight levels of reflection at port 3 of the coupler and and the input to the loop-matching network. In terms of dB (power), the reflection levels are as follows.

Parameter	Optimum Feedback	Weak feedback	No feedback
a_d	1.198	0	0
b_L	1.201	0	0
$20\log(a_d/b_d)$	-22.9 dB		
$20\log(b_L/a_L)$	-22.9 dB		

The incident and reflected waves at the coupler ports are included in Appendix H. The data show cancellation at port 2 of the coupler and -22.9 dB reflection at the input (port 1) of the coupler. The frequency response of the wave amplitude delivered to the load on port 2 of the coupler are also included. The bandwidth of the response is extremely narrow (approximately 40 KHz). The efficiency of the antenna array can be computed (since both the matching networks and antenna conductors are assumed lossless) as

$$\text{Prad/Pin} = 1-(.0172)^2-(.001)^2/\ 1-(.0172)^2 = 100\%$$

The corresponding efficiency bandwidth product is given by

$$\text{Eff}(\%) \times \text{BW}(\%) = 1.00*(.040/500) = 8.0 \times 10^{-5}$$

which is about 5.7 times smaller than the Wheeler limit of 4.55×10^{-4} calculated.

Case IV. Analysis With Optimum Feedback, Using Predicted Coupling and Added Phase Shift Values

The feedback parameters for Case IV are as follows.

Parameter	Predicted
Coupling factor, k	0.99788
Added differential phase shift, θ	63.41

Appendix I contains the incident and reflected wave values for Case IV. In general the cancellation at port 2 of the coupler was degraded (-16.2 dB vs. -58.7 dB for the Touchstone optimized feedback in Case III). The input match (reflection coefficient) was improved slightly (-23.1 dB versus -22.9 dB for Case III.) Due to the increased power in the load at port 2 of the coupler, the radiation efficiency for Case IV was computed as

$$\text{Prad/Pin} = 1\text{-}(.070)^2\text{-}(.146)^2/\ 1\text{-}(.070)^2 = 97.9\%$$

Appendix J contains the derivation of a Z-matrix for a tee-section and the frequency response for Case IV. The incomplete cancellation at port 2 is evident by noting the value at 500 MHz.

SIMULATION CONCLUSIONS

Clearly, high efficiencies can be obtained using feedback-matching techniques. However, if high efficiencies are desired, the ever-present bandwidth reduction trade-off is observed. Whether the mixed-mode antenna array follows the Wheeler limit has not yet been determined. Preliminary calculations for the mixed-mode array show that (1) without feedback matching the Wheeler limit has been exceeded by a factor of 3, and (2) with optimum feedback a limit reduction factor of 5.7 is observed. Additional work is needed to verify these preliminary results.

It is well recognized that high-circulating currents arise between the matching network and an isolated electrically small element. From a wave perspective, large incident and reflected waves are found at the matching network and antenna interfaces. For a single isolated antenna, the incident and reflected waves are present in the same transmission line. Impedance matching is accomplished by zeroing the wave incident upon the external load (or generator).

A parallel situation arises for the mixed-mode array with feedback matching. While incident and reflected waves are still present between the matching network and elements (i.e., loop and dipole), incident and reflected waves to and from the matched array are also present. Indeed, as was discovered in the simulations, the amplitudes of these feedback waves can become quite large. Additional work needs to be conducted in comparing the amplitudes of the feedback waves to the amplitudes of the waves between matching network and antenna elements.

Using the wave perspective, an additional degree of freedom appears to have been introduced that can be characterized by an intermediate matching-network impedance level. No longer are antenna element-matching networks restricted to output levels of 50 ohms but rather can be designed for arbitrary intermediate levels, perhaps even a complex value. Intuitively, to minimize energy storage in both the antenna and matching networks, the complementary nature of the mixed-mode antenna elements (i.e., inductance and capacitance) could be used to reduce the level of reactance needed for overall matching. In the extreme case, perhaps mutually resonant antenna elements with equal and opposite reactances could be designed. However, it may well turn out that since the matching networks considered here are all lossless, the best we can hope for is the bandwidth, which corresponds to the ratio of energy stored in the near field of both antennas to the energy radiated by both antennas. Much additional theoretical and empirical work is needed to assess the efficiency bandwidth performance of this array compared to the Wheeler limit.

CONCLUSIONS

The whole process can be broken down into simple steps. Given the (Z) matrix of the array and the antenna element currents, or their ratios, find the active impedance of each input port (antenna element). Design the "appropriate" two-port lossless-matching networks for each of these active impedances. Design the appropriate power divider/combiner that will give the correct currents at each element in amplitude and phase.

In Figure 7, a four-port directional coupler was chosen as the power combiner, arranged so that one port is decoupled for the proper excitations needed to give the correct antenna currents. A directional coupler is particularly simple but that does not mean other configurations cannot be used. In Figure 4, for example, just by drawing a box as the power-divider and two matching circuits, a universal three port is created.

Although the mathematics would be complicated, we could have tried to specify the s-parameters of this three port in such a fashion as to produce the proper currents, I_1 and I_2, be lossless, and have zero overall reflection. Once we had an appropriate set of s-parameters, we could have tried to design an appropriate three-port whose implementation might have been quite different than would naturally be arrived at using the approach achieved.

In any case, the virtue of active impedance matching as outlined in this report is that it breaks the problem into simple steps that are reasonably easily implemented.

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Appendix A

Y-PARAMETERS FOR MIXED-MODE ARRAY (Y-RECIPROCITY NOT ENFORCED)

MIXMODE2.S2P

```
! Simple Dipole Antenna located on Z-axis, center fed
! Dipole length = .02 wavelengths at 500 MHz
! Dipole radius = .001 wavelengths at 500 MHz
! Square loop in yz plane with side length = .025 wavelengths at 5
00 MHz
# GHZ Y RI R 1 !required
! data unmodified from NEC3D
!F(GHz) Y11r     Y11i       Y21r        Y21i        Y12r       Y12i
  Y22r       Y2
.45 7.4255E-10 3.0686E-4 4.9090E-9 -5.2599E-5 5.0949E-9 -5.0303E-5
 1.1422E-6 -1.1622E-2
.46 8.1044E-10 3.1370E-4 5.3603E-9 -5.3785E-5 5.5636E-9 -5.1437E-5
 1.1949E-6 -1.1355E-2
.47 8.8286E-10 3.2055E-4 5.8421E-9 -5.4973E-5 6.0639E-9 -5.2572E-5
 1.2489E-6 -1.1098E-2
.48 9.6001E-10 3.2740E-4 6.3556E-9 -5.6161E-5 6.5972E-9 -5.3708E-5
 1.3042E-6 -1.0852E-2
.49 1.0421E-09 3.3425E-4 6.9023E-9 -5.7350E-5 7.1650E-9 -5.4845E-5
 1.3608E-6 -1.0616E-2
.50 1.1293E-09 3.4111E-4 7.4836E-9 -5.8541E-5 7.7687E-9 -5.5983E-5
 1.4187E-6 -1.0389E-2
.51 1.2218E-09 3.4796E-4 8.1008E-9 -5.9733E-5 8.4098E-9 -5.7123E-5
 1.4779E-6 -1.0170E-2
.52 1.3198E-09 3.5482E-4 8.7554E-9 -6.0927E-5 9.0899E-9 -5.8264E-5
 1.5385E-6 -9.9601E-3
.53 1.4236E-09 3.6168E-4 9.4490E-9 -6.2121E-5 9.8105E-9 -5.9405E-5
 1.6004E-6 -9.7574E-3
.54 1.5334E-09 3.6854E-4 1.0183E-8 -6.3317E-5 1.0573E-8 -6.0549E-5
 1.6636E-6 -9.5619E-3
.55 1.6493E-09 3.7541E-4 1.0959E-8 -6.4514E-5 1.1379E-8 -6.1693E-5
 1.7282E-6 -9.3732E-3
```

Appendix B

Y-PARAMETERS FOR MIXED-MODE ARRAY (Y-RECIPROCITY ENFORCED)

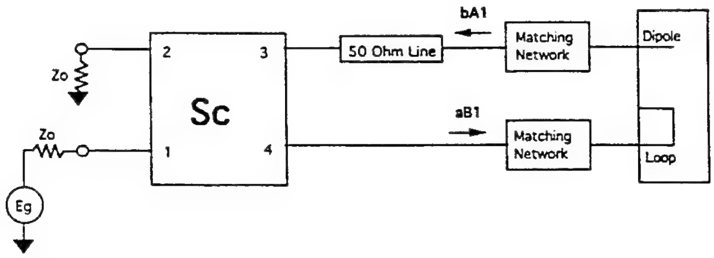
MIXMODE3.S2P

```
! Simple Dipole Antenna located on Z-axis, center fed
! Dipole length = .02 wavelengths at 500 MHz
! Dipole radius = .001 wavelengths at 500 MHz
! Square loop in yz plane with side length = .025 wavelengths at 5
00 MHz
# GHZ Y RI R 1 !required
! Y21 and Y12 values set to average value
!F(GHz) Y11r     Y11i       Y21r        Y21i       Y12r       Y12i
  Y22r       Y2
.45 7.4255E-10 3.0686E-4 5.0020E-9 -5.1451E-5 5.0020E-9 -5.1451E-5
 1.1422E-6 -1.1622E-2
.46 8.1044E-10 3.1370E-4 5.4620E-9 -5.2611E-5 5.4620E-9 -5.2611E-5
 1.1949E-6 -1.1355E-2
.47 8.8286E-10 3.2055E-4 5.9530E-9 -5.3773E-5 5.9530E-9 -5.3773E-5
 1.2489E-6 -1.1098E-2
.48 9.6001E-10 3.2740E-4 6.4764E-9 -5.4935E-5 6.4764E-9 -5.4935E-5
 1.3042E-6 -1.0852E-2
.49 1.0421E-09 3.3425E-4 7.0337E-9 -5.6098E-5 7.0337E-9 -5.6098E-5
 1.3608E-6 -1.0616E-2
.50 1.1293E-09 3.4111E-4 7.6262E-9 -5.7262E-5 7.6262E-9 -5.7262E-5
 1.4187E-6 -1.0389E-2
.51 1.2218E-09 3.4796E-4 8.2553E-9 -5.8428E-5 8.2553E-9 -5.8428E-5
 1.4779E-6 -1.0170E-2
.52 1.3198E-09 3.5482E-4 8.9227E-9 -5.9596E-5 8.9227E-9 -5.9596E-5
 1.5385E-6 -9.9601E-3
.53 1.4236E-09 3.6168E-4 9.6298E-9 -6.0763E-5 9.6298E-9 -6.0763E-5
 1.6004E-6 -9.7574E-3
.54 1.5334E-09 3.6854E-4 1.0378E-8 -6.1933E-5 1.0378E-8 -6.1933E-5
 1.6636E-6 -9.5619E-3
.55 1.6493E-09 3.7541E-4 1.1169E-8 -6.3104E-5 1.1169E-8 -6.3104E-5
 1.7282E-6 -9.3732E-3
```

Appendix C

MATHCAD ANALYSIS FOR MIXED-MODE ARRAY (Y-RECIPROCITY NOT ENFORCED)

Consider the following mixed-mode antenna with feedback matching



Antenna Y-matrix reciprocity not enforced

Read in the Y-matrix for the mixed-mode antenna. Port 1 is the dipole and port 2 is the loop.

$Y2P = READPRN(mixmodel)$ $\qquad$ $m = 1..11$

$j = 1..2$

$k = 1..2$

$Freq_m = 450\cdot 10^6 + (M-1)\cdot 10\cdot 10^6$

$Y450_{j,k} = Y2P_{1\cdot j.4+k\cdot 2-5} + Y2P_{1,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z450 = Y450^{-1}$

$Y460_{j,k} = Y2P_{2\cdot j.4+k\cdot 2-5} + Y2P_{2,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z460 = Y460^{-1}$

$Y470_{j,k} = Y2P_{3\cdot j.4+k\cdot 2-5} + Y2P_{3,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z470 = Y470^{-1}$

$Y480_{j,k} = Y2P_{4\cdot j.4+k\cdot 2-5} + Y2P_{4,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z480 = Y480^{-1}$

$Y490_{j,k} = Y2P_{5\cdot j.4+k\cdot 2-5} + Y2P_{5,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z490 = Y490^{-1}$

$Y500_{j,k} = Y2P_{6\cdot j.4+k\cdot 2-5} + Y2P_{6,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z500 = Y500^{-1}$

$Y510_{j,k} = Y2P_{7\cdot j.4+k\cdot 2-5} + Y2P_{7,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z510 = Y510^{-1}$

$Y520_{j,k} = Y2P_{8\cdot j.4+k\cdot 2-5} + Y2P_{8,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z520 = Y520^{-1}$

$Y530_{j,k} = Y2P_{9\cdot j.4+k\cdot 2-5} + Y2P_{9,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z530 = Y530^{-1}$

$Y540_{j,k} = Y2P_{10\cdot j.4+k\cdot 2-5} + Y2P_{10,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z540 = Y540^{-1}$

$Y550_{j,k} = Y2P_{11\cdot j.4+k\cdot 2-5} + Y2P_{11,j\cdot 4+k\cdot 2-4}\cdot i$ $\qquad$ $Z550 = Y550^{-1}$

DESCRIPTION OF MIXED-MODE ARRAY

Mixed-mode antenna is comprised of a center-fed dipole (along the z-axis) whose length is 0.020λ and whose wire radius is 0.001 λ. Surrounding the dipole (in the y-z plane) is a square loop with side length of 0.025λ and wire radius of 0.001λ. The loop is fed at the point where x = 0, y = 0.01251, and z = 0. This mixed-mode antenna is modelled using NEC3D. The dipole is modelled using five segments and the loop is modelled using five segments per side. The extended kernal is used. Lossless conductors are assumed.

Compute wavelength in inches

$$\lambda inch_m = \frac{2.997925 \cdot 10^{10}}{2.54 \cdot Freq_m}$$

Enter physical length of dipole (inches)

Ldipole = 0.472114173

Enter side length of square loop (inches)

Lloop = 0.590142717

Normalize to wavelength

$$Ld_m = \frac{Ldipole}{\lambda inch_m} \qquad Freq\_MHz_m = \frac{Freq_m}{10^6}$$

$$Sloop_m = \frac{Lloop}{\lambda inch_m}$$

$$ILoverId_m = \frac{Ld_m}{2 \cdot \pi \cdot (Sloop_m)^2}$$

Dipole current

$$Iin_{m.1} = 1 \cdot \exp\left(-j \cdot 90 \frac{\pi}{180}\right)$$

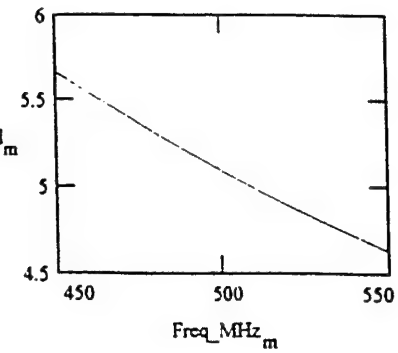
$$Iin_{1.1} = -i$$

Loop current

$$Iin_{m.2} = ILoverId_m$$

$$Iin_{1,2} = 5.65884$$

Required Current Ratio as Function of Frequency



Calculate the active impedance

$$j = 1..2$$

$$Z450a_j = Z450_{j,1} \cdot \frac{Iin_{1,1}}{Iin_{1,j}} + Z450_{j,2} \cdot \frac{Iin_{1,2}}{Iin_{1,j}} \quad Z450a = \begin{pmatrix} -83.39155 - 3.2564 \cdot 10^3 i \\ 2.49917 + 85.97993i \end{pmatrix}$$

$$Z460a_j = Z460_{j,1} \cdot \frac{Iin_{2,1}}{Iin_{2,j}} + Z460_{j,2} \cdot \frac{Iin_{2,2}}{Iin_{2,j}} \quad Z460a = \begin{pmatrix} -83.51475 - 3.18528 \cdot 10^3 i \\ 2.61575 + 85.9986i \end{pmatrix}$$

$$Z470a_j = Z470_{j,1} \cdot \frac{Iin_{3,1}}{Iin_{3,j}} + Z470_{j,2} \cdot \frac{Iin_{3,2}}{Iin_{3,j}} \quad Z470a = \begin{pmatrix} -83.64797 - 3.11711 \cdot 10^3 i \\ 2.73547 + 90.0332i \end{pmatrix}$$

$$Z480a_j = Z480_{j,1} \cdot \frac{Iin_{4,1}}{Iin_{4,j}} + Z480_{j,2} \cdot \frac{Iin_{4,2}}{Iin_{4,j}} \quad Z480a = \begin{pmatrix} -83.77837 - 3.05178 \cdot 10^3 i \\ 2.85805 + 92.07076i \end{pmatrix}$$

$$Z490a_j = Z490_{j,1} \cdot \frac{Iin_{5,1}}{Iin_{5,j}} + Z490_{j,2} \cdot \frac{Iin_{5,2}}{Iin_{5,j}} \quad Z490a = \begin{pmatrix} -83.91002 - 2.98912 \cdot 10^3 i \\ 2.98358 + 94.11403i \end{pmatrix}$$

$$Z500a_j = Z500_{j,1} \cdot \frac{Iin_{6,1}}{Iin_{6,j}} + Z500_{j,2} \cdot \frac{Iin_{6,2}}{Iin_{6,j}} \quad Z500a = \begin{pmatrix} -83.04509 - 2.9289 \cdot 10^3 i \\ 3.11211 + 96.16673i \end{pmatrix}$$

$$Z510a_j = Z510_{j,1} \cdot \frac{Iin_{7,1}}{Iin7_{2,j}} + Z510_{j,2} \cdot \frac{Iin_{7,2}}{Iin_{7,j}} \quad Z510a = \begin{pmatrix} -84.19096 - 2.87112 \cdot 10^3 i \\ 3.24406 + 98.23371i \end{pmatrix}$$

$$Z520a_j = Z520_{j,1} \cdot \frac{Iin_{87,1}}{Iin_{8,j}} + Z520_{j,2} \cdot \frac{Iin_{8,2}}{Iin_{8,j}} \quad Z520a = \begin{pmatrix} -84.33095 - 2.8155 \cdot 10^3 i \\ 3.37873 + 100.29986i \end{pmatrix}$$

$$Z530a_j = Z530_{j,1} \cdot \frac{Iin_{9,1}}{Iin_{9,j}} + Z530_{j,2} \cdot \frac{Iin_{9,2}}{Iin_{9,j}} \quad Z530a = \begin{pmatrix} -84.47664 - 2.76199 \cdot 10^3 i \\ 3.51663 + 102.37928i \end{pmatrix}$$

$$Z540a_j = Z540_{j,1} \cdot \frac{Iin_{10,1}}{Iin_{10,j}} + Z540_{j,2} \cdot \frac{Iin_{10,2}}{Iin_{10,j}} \quad Z540a = \begin{pmatrix} -84.62726 - 2.71046 \cdot 10^3 i \\ 3.65784 + 104.46809i \end{pmatrix}$$

$$Z550a_j = Z550_{j,1} \cdot \frac{Iin_{11,1}}{Iin_{11,j}} + Z550_{j,2} \cdot \frac{Iin_{11,2}}{Iin_{11,j}} \quad Z550a = \begin{pmatrix} -84.77924 - 2.66074 \cdot 10^3 i \\ 3.80212 + 106.5663i \end{pmatrix}$$

Put active impedances into array

$$\begin{array}{cc} & \text{Dipole} \qquad\qquad\qquad\qquad \text{Loop} \\ Za = & \begin{bmatrix} \mathrm{Re}(Z450a_1) & \mathrm{Im}(Z450a_1) & \mathrm{Re}(Z450a_2) & \mathrm{Im}(Z450a_2) \\ \mathrm{Re}(Z460a_1) & \mathrm{Im}(Z460a_1) & \mathrm{Re}(Z460a_2) & \mathrm{Im}(Z460a_2) \\ \mathrm{Re}(Z470a_1) & \mathrm{Im}(Z470a_1) & \mathrm{Re}(Z470a_2) & \mathrm{Im}(Z470a_2) \\ \mathrm{Re}(Z480a_1) & \mathrm{Im}(Z480a_1) & \mathrm{Re}(Z480a_2) & \mathrm{Im}(Z480a_2) \\ \mathrm{Re}(Z490a_1) & \mathrm{Im}(Z490a_1) & \mathrm{Re}(Z490a_2) & \mathrm{Im}(Z490a_2) \\ \mathrm{Re}(Z500a_1) & \mathrm{Im}(Z500a_1) & \mathrm{Re}(Z500a_2) & \mathrm{Im}(Z500a_2) \\ \mathrm{Re}(Z510a_1) & \mathrm{Im}(Z510a_1) & \mathrm{Re}(Z510a_2) & \mathrm{Im}(Z510a_2) \\ \mathrm{Re}(Z520a_1) & \mathrm{Im}(Z520a_1) & \mathrm{Re}(Z520a_2) & \mathrm{Im}(Z520a_2) \\ \mathrm{Re}(Z530a_1) & \mathrm{Im}(Z530a_1) & \mathrm{Re}(Z530a_2) & \mathrm{Im}(Z530a_2) \\ \mathrm{Re}(Z540a_1) & \mathrm{Im}(Z540a_1) & \mathrm{Re}(Z540a_2) & \mathrm{Im}(Z540a_2) \\ \mathrm{Re}(Z550a_1) & \mathrm{Im}(Z550a_1) & \mathrm{Re}(Z550a_2) & \mathrm{Im}(Z550a_2) \end{bmatrix} \end{array}$$

Active impendances at 500 MHz

$$Z500a = \begin{pmatrix} -84.04509 - 2.9289 \cdot 10^3 i \\ 3.11211 + 96.16673i \end{pmatrix} \begin{matrix} \mathit{Dipole} \\ \mathit{Loop} \end{matrix}$$

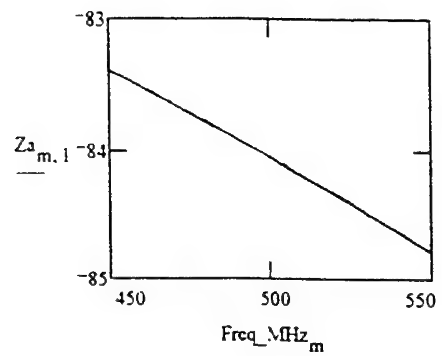
Z-matrix

$$Z500 = \begin{pmatrix} 0.00934 - 2.9289 \cdot 10^3 i & 9.13542 \cdot 10^{-5} + 16.50405i \\ -8.52033 \cdot 10^{-5} + 15.78289i & 0.01314 + 96.16672i \end{pmatrix}$$

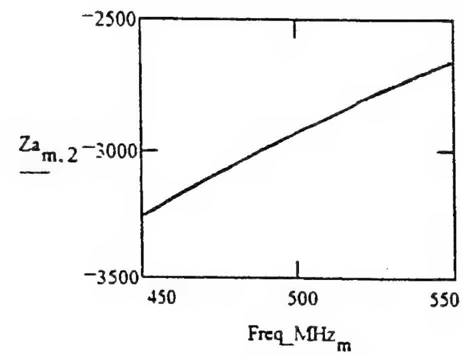
Y-matrix

$$Y500 = \begin{pmatrix} 1.1293\cdot 10^{-9} + 3.4111\cdot 10^{-4} i & 7.4836\cdot 10^{-9} - 5.8541\cdot 10^{-5} i \\ 7.7687\cdot 10^{-9} - 5.5983\cdot 10^{-5} i & 1.4187\cdot 10^{-6} - 0.01039 i \end{pmatrix}$$

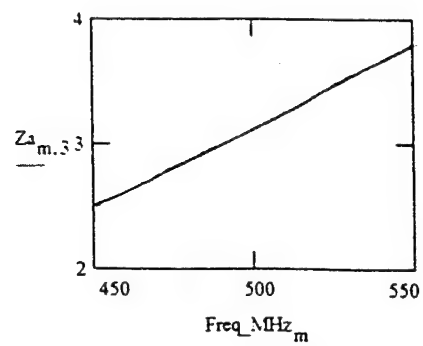
Dipole Resistance



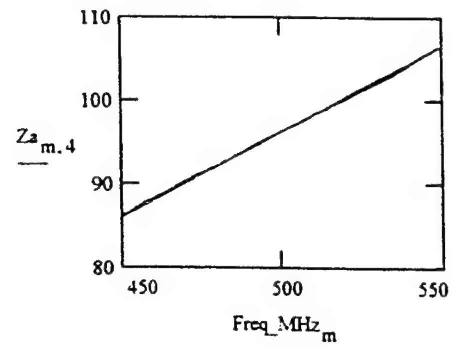
Dipole Reactance



Loop Resistance



Loop Reactance



Next, calculate the scattering parameters of the required matched networks

$Z0 = 50$

Matching network for positive R (S22 = Γi*)

$$S22loop_m = \overline{\left[\mathrm{Re}\left[\frac{(Za_{m,3} - Z0) + Za_{m,4} \cdot i}{(Za_{m,3} - Z0) + Za_{m,4} \cdot i}\right] + i \cdot \mathrm{Im}\left[\frac{(Za_{m,3} - Z0) + Za_{m,4} \cdot i}{(Za_{m,3} - Z0) + Za_{m,4} \cdot i}\right]\right]}$$

$$S22loopmag_m = |S22loop_m| \qquad S22loop_6 = 0.55993 - 0.79681i$$

$$S22loopang_m = \arg(S22loop_m) \cdot \frac{180}{\pi} \qquad S22loopmag_6 = 0.97387$$

$$S22loopang_6 = 54.90393$$

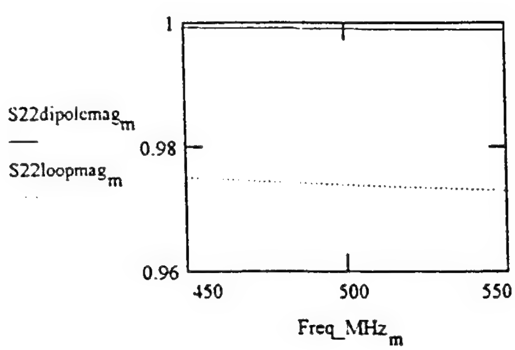
Matching network for negative R (S22 = 1/Γi)

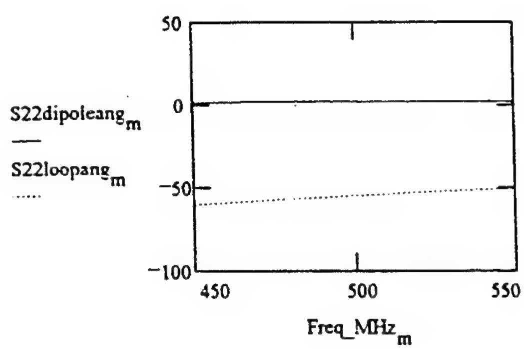
$$S22dipole_m = \mathrm{Re}\left(\frac{Za_{m,1} + Z0 + Za_{m,2} \cdot i}{Za_{m,1} - Z0 + Za_{m,2} \cdot i}\right) + i \cdot \mathrm{Im}\left(\frac{Za_{m,1} + Z0 + Za_{m,2} \cdot i}{Za_{m,1} - Z0 + Za_{m,2} \cdot i}\right)$$

$$S22dipolemag_m = |S22dipole_m| \qquad S22dipole_6 = 0.99844 + 0.03407ii$$

$$S22dipoleang_m = \arg(S22dipole_m) \cdot \frac{180}{\pi} \qquad S22dipolemag_6 = 0.99902$$

$$S22dipoleang_6 = 1.95443$$





Calculate S12 for matching network of both dipole and loop

$$S12dipolemag_m = \sqrt{1-(S22dipolemag_m)2}$$

$$S12loopmag_m = \sqrt{1-(S22loopmag_m)2}$$

$$S12dipoleang_m = S22dipoleang_m - 90$$

$$S12loopang_m = S22loopang_m - 90$$

Formulate matching network S-matrix for dipole

$$Sd450_{1,1}\, S22dipolemag_1 \cdot e^{(j \cdot S22dipoleang_1)\cdot\frac{\pi}{180}} \qquad Sd450_{2,2}\, S22dipolemag_1 \cdot e^{(j \cdot S22dipoleang_1)\cdot\frac{\pi}{180}}$$

$$Sd450_{1,2}\, S12dipolemag_1 \cdot e^{(j \cdot S12dipoleang_1)\cdot\frac{\pi}{180}} \qquad Sd450_{2,1}\, S12dipolemag_1 \cdot e^{(j \cdot S12dipoleang_1)\cdot\frac{\pi}{180}}$$

$$Sd460_{1,1}\, S22dipolemag_2 \cdot e^{(j \cdot S22dipoleang_2)\cdot\frac{\pi}{180}} \qquad Sd460_{2,2}\, S22dipolemag_2 \cdot e^{(j \cdot S22dipoleang_2)\cdot\frac{\pi}{180}}$$

$$Sd460_{1,2}\, S22dipolemag_2 \cdot e^{(j \cdot S12dipoleang_2)\cdot\frac{\pi}{180}} \qquad Sd460_{2,1}\, S12dipolemag_2 \cdot e^{(j \cdot S12dipoleang_2)\cdot\frac{\pi}{180}}$$

$$Sd470_{1,1}\, S22dipolemag_3 \cdot e^{(j \cdot S22dipoleang_3)\cdot\frac{\pi}{180}} \qquad Sd470_{2,2}\, S22dipolemag_3 \cdot e^{(j \cdot S22dipoleang_3)\cdot\frac{\pi}{180}}$$

$$Sd470_{1,2}\, S12dipolemag_3 \cdot e^{(j \cdot S12dipoleang_3)\cdot\frac{\pi}{180}} \qquad Sd470_{2,1}\, S12dipolemag_3 \cdot e^{(j \cdot S12dipoleang_3)\cdot\frac{\pi}{180}}$$

$$Sd480_{1,1}\, S22dipolemag_4 \cdot e^{(j \cdot S22dipoleang_4)\cdot\frac{\pi}{180}} \qquad Sd480_{2,2}\, S22dipolemag_4 \cdot e^{(j \cdot S22dipoleang_4)\cdot\frac{\pi}{180}}$$

$$Sd480_{1,2}\, S12dipolemag_4 \cdot e^{(j \cdot S12dipoleang_4)\cdot\frac{\pi}{180}} \qquad Sd480_{2,1}\, S12dipolemag_4 \cdot e^{(j \cdot S12dipoleang_4)\cdot\frac{\pi}{180}}$$

$$Sd490_{1,1}\, S22dipolemag_5 \cdot e^{(j \cdot S22dipoleang_5)\cdot\frac{\pi}{180}} \qquad Sd490_{2,2}\, S22dipolemag_5 \cdot e^{(j \cdot S22dipoleang_5)\cdot\frac{\pi}{180}}$$

$$Sd490_{1,2}\, S12dipolemag_5 \cdot e^{(j \cdot S12dipoleang_5)\cdot\frac{\pi}{180}} \qquad Sd490_{2,1}\, S12dipolemag_5 \cdot e^{(j \cdot S12dipoleang_5)\cdot\frac{\pi}{180}}$$

$$Sd500_{1,1}\, S22dipolemag_6 \cdot e^{(j \cdot S22dipoleang_6)\cdot\frac{\pi}{180}} \qquad Sd500_{2,2}\, S22dipolemag_6 \cdot e^{(j \cdot S22dipoleang_6)\cdot\frac{\pi}{180}}$$

$$Sd500_{1,2}\, S12dipolemag_6 \cdot e^{(j \cdot S12dipoleang_6)\cdot\frac{\pi}{180}} \qquad Sd500_{2,1}\, S12dipolemag_6 \cdot e^{(j \cdot S12dipoleang_6)\cdot\frac{\pi}{180}}$$

$$Sd510_{1,1}\,S22dipolemag_7 \cdot e^{\left(j \cdot S22dipoleang_7\right) \cdot \frac{\pi}{180}} \qquad Sd510_{2,2}\,S22dipolemag_7 \cdot e^{\left(j \cdot S22dipoleang_7\right) \cdot \frac{\pi}{180}}$$

$$Sd510_{1,2}\,S12dipolemag_7 \cdot e^{\left(j \cdot S12dipoleang_7\right) \cdot \frac{\pi}{180}} \qquad Sd510_{2,1}\,S12dipolemag_7 \cdot e^{\left(j \cdot S12dipoleang_7\right) \cdot \frac{\pi}{180}}$$

$$Sd520_{1,1}\,S22dipolemag_8 \cdot e^{\left(j \cdot S22dipoleang_8\right) \cdot \frac{\pi}{180}} \qquad Sd520_{2,2}\,S22dipolemag_8 \cdot e^{\left(j \cdot S22dipoleang_8\right) \cdot \frac{\pi}{180}}$$

$$Sd520_{1,2}\,S12dipolemag_8 \cdot e^{\left(j \cdot S12dipoleang_8\right) \cdot \frac{\pi}{180}} \qquad Sd520_{2,1}\,S12dipolemag_8 \cdot e^{\left(j \cdot S12dipoleang_8\right) \cdot \frac{\pi}{180}}$$

$$Sd530_{1,1}\,S22dipolemag_9 \cdot e^{\left(j \cdot S22dipoleang_9\right) \cdot \frac{\pi}{180}} \qquad Sd530_{2,2}\,S22dipolemag_9 \cdot e^{\left(j \cdot S22dipoleang_9\right) \cdot \frac{\pi}{180}}$$

$$Sd530_{1,2}\,S12dipolemag_9 \cdot e^{\left(j \cdot S12dipoleang_9\right) \cdot \frac{\pi}{180}} \qquad Sd530_{2,1}\,S12dipolemag_9 \cdot e^{\left(j \cdot S12dipoleang_9\right) \cdot \frac{\pi}{180}}$$

$$Sd540_{1,1}\,S22dipolemag_{10} \cdot e^{\left(j \cdot S22dipoleang_{10}\right) \cdot \frac{\pi}{180}} \qquad Sd540_{2,2}\,S22dipolemag_{10} \cdot e^{\left(j \cdot S22dipoleang_{10}\right) \cdot \frac{\pi}{180}}$$

$$Sd540_{1,2}\,S12dipolemag_{10} \cdot e^{\left(j \cdot S12dipoleang_{10}\right) \cdot \frac{\pi}{180}} \qquad Sd540_{2,1}\,S12dipolemag_{10} \cdot e^{\left(j \cdot S12dipoleang_{10}\right) \cdot \frac{\pi}{180}}$$

$$Sd550_{1,1}\,S22dipolemag_{11} \cdot e^{\left(j \cdot S22dipoleang_{11}\right) \cdot \frac{\pi}{180}} \qquad Sd550_{2,2}\,S22dipolemag_{11} \cdot e^{\left(j \cdot S22dipoleang_{11}\right) \cdot \frac{\pi}{180}}$$

$$Sd550_{1,2}\,S12dipolemag_{11} \cdot e^{\left(j \cdot S12dipoleang_{11}\right) \cdot \frac{\pi}{180}} \qquad Sd550_{2,1}\,S12dipolemag_{11} \cdot e^{\left(j \cdot S12dipoleang_{11}\right) \cdot \frac{\pi}{180}}$$

Formulate matching network S-matrix for loop

$$Sl450_{1,1}\,S22loopmag_1 \cdot e^{\left(j \cdot S22loopang_1\right) \cdot \frac{\pi}{180}} \qquad Sl450_{2,2}\,S22loopmag_1 \cdot e^{\left(j \cdot S22loopang_1\right) \cdot \frac{\pi}{180}}$$

$$Sl450_{1,2}\,S12loopmag_1 \cdot e^{\left(j \cdot S12loopang_1\right) \cdot \frac{\pi}{180}} \qquad Sl450_{2,1}\,S12loopmag_1 \cdot e^{\left(j \cdot S12loopang_1\right) \cdot \frac{\pi}{180}}$$

$$Sl460_{1,1}\,S22loopmag_2 \cdot e^{\left(j \cdot S22loopang_2\right) \cdot \frac{\pi}{180}} \qquad Sl460_{2,2}\,S22loopmag_2 \cdot e^{\left(j \cdot S22loopang_2\right) \cdot \frac{\pi}{180}}$$

$$Sl460_{1,2}\,S22loopmag_2 \cdot e^{\left(j \cdot S12loopang_2\right) \cdot \frac{\pi}{180}} \qquad Sl460_{2,1}\,S12loopmag_2 \cdot e^{\left(j \cdot S12loopang_2\right) \cdot \frac{\pi}{180}}$$

$$S1470_{1,1}\,S22loopmag_{3}\cdot e^{\left(j\cdot S22loopang_{3}\right)\cdot\frac{\pi}{180}} \qquad S1470_{2,2}\,S22loopmag_{3}\cdot e^{\left(j\cdot S22loopang_{3}\right)\cdot\frac{\pi}{180}}$$

$$S1470_{1,2}\,S12loopmag_{3}\cdot e^{\left(j\cdot S12loopang_{3}\right)\cdot\frac{\pi}{180}} \qquad S1470_{2,1}\,S12loopmag_{3}\cdot e^{\left(j\cdot S12loopang_{3}\right)\cdot\frac{\pi}{180}}$$

$$S1480_{1,1}\,S22loopmag_{4}\cdot e^{\left(j\cdot S22loopang_{4}\right)\cdot\frac{\pi}{180}} \qquad S1480_{2,2}\,S22loopmag_{4}\cdot e^{\left(j\cdot S22loopeang_{4}\right)\cdot\frac{\pi}{180}}$$

$$S1480_{1,2}\,S12loopmag_{4}\cdot e^{\left(j\cdot S12loopang_{4}\right)\cdot\frac{\pi}{180}} \qquad S1480_{2,1}\,S12loopmag_{4}\cdot e^{\left(j\cdot S12loopang_{4}\right)\cdot\frac{\pi}{180}}$$

$$S1490_{1,1}\,S22loopmag_{5}\cdot e^{\left(j\cdot S22loopang_{5}\right)\cdot\frac{\pi}{180}} \qquad S1490_{2,2}\,S22loopmag_{5}\cdot e^{\left(j\cdot S22loopang_{5}\right)\cdot\frac{\pi}{180}}$$

$$S1490_{1,2}\,S12loopmag_{5}\cdot e^{\left(j\cdot S12loopang_{5}\right)\cdot\frac{\pi}{180}} \qquad S1490_{2,1}\,S12loopmag_{5}\cdot e^{\left(j\cdot S12loopang_{5}\right)\cdot\frac{\pi}{180}}$$

$$S1500_{1,1}\,S22loopmag_{6}\cdot e^{\left(j\cdot S22loopang_{6}\right)\cdot\frac{\pi}{180}} \qquad S1500_{2,2}\,S22loopmag_{6}\cdot e^{\left(j\cdot S22loopang_{6}\right)\cdot\frac{\pi}{180}}$$

$$S1500_{1,2}\,S12loopmag_{6}\cdot e^{\left(j\cdot S12loopang_{6}\right)\cdot\frac{\pi}{180}} \qquad S1500_{2,1}\,S12loopmag_{6}\cdot e^{\left(j\cdot S12loopang_{6}\right)\cdot\frac{\pi}{180}}$$

$$S1510_{1,1}\,S22loopmag_{7}\cdot e^{\left(j\cdot S22loopang_{7}\right)\cdot\frac{\pi}{180}} \qquad S1510_{1,1}\,S22loopmag_{7}\cdot e^{\left(j\cdot S22loopang_{7}\right)\cdot\frac{\pi}{180}}$$

$$S1510_{1,2}\,S12loopmag_{7}\cdot e^{\left(j\cdot S12loopang_{7}\right)\cdot\frac{\pi}{180}} \qquad S1510_{2,1}\,S12loopmag_{7}\cdot e^{\left(j\cdot S12loopang_{7}\right)\cdot\frac{\pi}{180}}$$

$$S1520_{1,1}\,S22loopmag_{8}\cdot e^{\left(j\cdot S22loopang_{8}\right)\cdot\frac{\pi}{180}} \qquad S1520_{2,2}\,S22loopmag_{8}\cdot e^{\left(j\cdot S22loopang_{8}\right)\cdot\frac{\pi}{180}}$$

$$S1520_{1,2}\,S12loopmag_{8}\cdot e^{\left(j\cdot S12loopang_{8}\right)\cdot\frac{\pi}{180}} \qquad S1520_{2,1}\,S12loopmag_{8}\cdot e^{\left(j\cdot S12loopang_{8}\right)\cdot\frac{\pi}{180}}$$

$$S1530_{1,1}\,S22loopmag_{9}\cdot e^{\left(j\cdot S22loopang_{9}\right)\cdot\frac{\pi}{180}} \qquad S1530_{1,1}\,S22loopmag_{9}\cdot e^{\left(j\cdot S22loopang_{9}\right)\cdot\frac{\pi}{180}}$$

$$S1530_{1,2}\,S12loopmag_{9}\cdot e^{\left(j\cdot S12loopang_{9}\right)\cdot\frac{\pi}{180}} \qquad S1530_{1,2}\,S12loopmag_{9}\cdot e^{\left(j\cdot S12loopang_{9}\right)\cdot\frac{\pi}{180}}$$

$$S1540_{1,1}\,S22loopmag_{10}\cdot e^{\left(j\cdot S22loopang_{10}\right)\cdot\frac{\pi}{180}} \qquad S1540_{2,2}\,S22loopmag_{10}\cdot e^{\left(j\cdot S22loopang_{10}\right)\cdot\frac{\pi}{180}}$$

$$S1540_{1,2}\,S12loopmag_{10}\cdot e^{\left(j\cdot S12loopang_{10}\right)\cdot\frac{\pi}{180}} \qquad S1540_{2,1}\,S12loopmag_{10}\cdot e^{\left(j\cdot S12loopang_{10}\right)\cdot\frac{\pi}{180}}$$

$$S1550_{1,1}\,S22loopmag_{11}\cdot e^{\left(j\cdot S22loopang_{11}\right)\cdot\frac{\pi}{180}} \qquad S1550_{2,2}\,S22loopmag_{11}\cdot e^{\left(j\cdot S22loopang_{11}\right)\cdot\frac{\pi}{180}}$$

$$S1550_{1,2}\,S12loopmag_{11}\cdot e^{\left(j\cdot S12loopang_{11}\right)\cdot\frac{\pi}{180}} \qquad S1550_{2,1}\,S12loopmag_{11}\cdot e^{\left(j\cdot S12loopang_{11}\right)\cdot\frac{\pi}{180}}$$

Find the corresponding Z-matrix

$$Sd500 = \begin{pmatrix} 0.99844 + 0.03407i & 0.00151 - 0.04419i \\ 0.00151 - 0.04419i & 0.99844 + 0.03407i \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad Z0 = 50 \qquad Sl500 = \begin{pmatrix} 0.55993 - 0.79681i & -0.18581 - 0.13057i \\ -0.18581 - 0.13057i & 0.55993 - 0.79681i \end{pmatrix}$$

$$Zd450 = (I - Sd450)^{-1} \cdot (I + Sd450) \cdot Z0 \quad Zl450 = (I - Sl450)^{-1} \cdot (I + Sl450) \cdot Z0$$
$$Zd460 = (I - Sd460)^{-1} \cdot (I + Sd460) \cdot Z0 \quad Zl460 = (I - Sl460)^{-1} \cdot (I + Sl460) \cdot Z0$$
$$Zd470 = (I - Sd470)^{-1} \cdot (I + Sd470) \cdot Z0 \quad Zl470 = (I - Sl470)^{-1} \cdot (I + Sl470) \cdot Z0$$
$$Zd480 = (I - Sd480)^{-1} \cdot (I + Sd480) \cdot Z0 \quad Zl480 = (I - Sl480)^{-1} \cdot (I + Sl480) \cdot Z0$$
$$Zd490 = (I - Sd490)^{-1} \cdot (I + Sd490) \cdot Z0 \quad Zl490 = (I - Sl490)^{-1} \cdot (I + Sl490) \cdot Z0$$
$$Zd500 = (I - Sd500)^{-1} \cdot (I + Sd500) \cdot Z0 \quad Zl500 = (I - Sl500)^{-1} \cdot (I + Sl500) \cdot Z0$$
$$Zd510 = (I - Sd510)^{-1} \cdot (I + Sd510) \cdot Z0 \quad Zl510 = (I - Sl510)^{-1} \cdot (I + Sl5120) \cdot Z0$$
$$Zd520 = (I - Sd520)^{-1} \cdot (I + Sd520) \cdot Z0 \quad Zl520 = (I - Sl520)^{-1} \cdot (I + Sl520) \cdot Z0$$
$$Zd530 = (I - Sd530)^{-1} \cdot (I + Sd530) \cdot Z0 \quad Zl530 = (I - Sl530)^{-1} \cdot (I + Sl530) \cdot Z0$$
$$Zd540 = (I - Sd540)^{-1} \cdot (I + Sd540) \cdot Z0 \quad Zl540 = (I - Sl540)^{-1} \cdot (I + Sl540) \cdot Z0$$
$$Zd550 = (I - Sd550)^{-1} \cdot (I + Sd550) \cdot Z0 \quad Zl550 = (I - Sl550)^{-1} \cdot (I + Sl550) \cdot Z0$$

Next, find elements of equivalent tee-network

$$Zd500 \begin{pmatrix} -4.3015 \cdot 10^3 i & -5.57725 \cdot 10^3 i \\ -5.57725 \cdot 10^3 i & -4.315 \cdot 10^3 i \end{pmatrix}$$

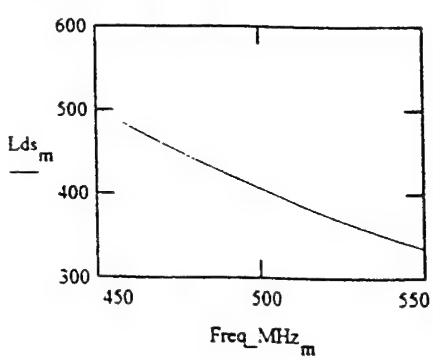
$$Zl500 \begin{pmatrix} -102.54964i & 28.46349i \\ 28.4639i & -102.54964i \end{pmatrix}$$

$$Zds_1 = Zd450_{1,1} - Zd450_{1,2} \qquad Zdp_1 = Zd450_{1,2}$$
$$Zds_2 = Zd460_{1,1} - Zd460_{1,2} \qquad Zdp_2 = Zd460_{1,2}$$
$$Zds_3 = Zd470_{1,1} - Zd470_{1,2} \qquad Zdp_3 = Zd470_{1,2}$$
$$Zds_4 = Zd480_{1,1} - Zd480_{1,2} \qquad Zdp_4 = Zd480_{1,2}$$
$$Zds_5 = Zd490_{1,1} - Zd490_{1,2} \qquad Zdp_5 = Zd490_{1,2}$$
$$Zds_6 = Zd500_{1,1} - Zd500_{1,2} \qquad Zdp_6 = Zd500_{1,2}$$
$$Zds_7 = Zd510_{1,1} - Zd510_{1,2} \qquad Zdp_7 = Zd510_{1,2}$$
$$Zds_8 = Zd520_{1,1} - Zd520_{1,2} \qquad Zdp_8 = Zd520_{1,2}$$
$$Zds_9 = Zd530_{1,1} - Zd530_{1,2} \qquad Zdp_9 = Zd530_{1,2}$$
$$Zds_{10} = Zd540_{1,1} - Zd540_{1,2} \qquad Zdp_{10} = Zd540_{1,2}$$
$$Zds_{11} = Zd550_{1,1} - Zd550_{1,2} \qquad Zdp_{11} = Zd550_{1,2}$$

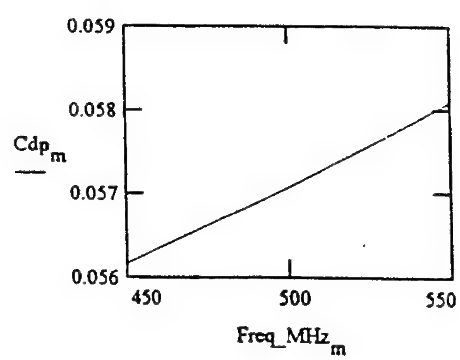
$$Lds_m = \frac{Zds_m}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6} \cdot 10^9$$

$$Cdp_m = \frac{1}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6 \cdot Zdp_m} \cdot 10^{12}$$

Dipole-Matching Network
Series Inductance vs. Frequency



Dipole-Matching Network
Parallel Capacitance vs. Frequency



$Zls_1 = Zl450_{1,1} - Zl450_{1,2}$ $Zlp_1 = Zl450_{1,2}$ $Zlp_1 = 23.11637i$

$Zls_2 = Zl460_{1,1} - Zl460_{1,2}$ $Zlp_2 = Zl460_{1,2}$ $Zlp_2 = 24.12184i$

$Zls_3 = Zl470_{1,1} - Zl470_{1,2}$ $Zlp_3 = Zl470_{1,2}$

$Zls_4 = Zl480_{1,1} - Zl480_{1,2}$ $Zlp_4 = Zl480_{1,2}$

$Zls5 = Zl490_{1,1} - Zl490_{1,2}$ $Zlp_5 = Zl490_{1,2}$

$Zls6 = Zl500_{1,1} - Zl500_{1,2}$ $Zlp6 = Zl5000_{1,2}$

$Zls_7 = Zl510_{1,1} - Zl510_{1,2}$ $Zlp_7 = Zl510_{1,2}$

$Zls_8 = Zl520_{1,1} - Zl520_{1,2}$ $Zlp_8 = Zl520_{1,2}$

$Zls9 = Zl530_{1,1} - Zl530_{1,2}$ $Zlp_9 = Zl530_{1,2}$

$Zls_{10} = Zl540_{1,1} - Zl540_{1,2}$ $Zlp10 = Zl540_{1,2}$

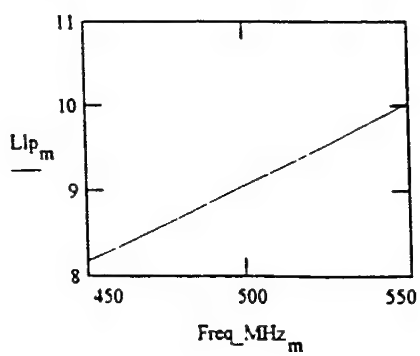
$Zls_{11} = Zl550_{1.1} - Zl550_{1,2}$ $Zlp_{11} = Zl550_{1,2}$

$Zls_6 = 131.0134i$ $Zlp_2 = Zl460_{1,2}$

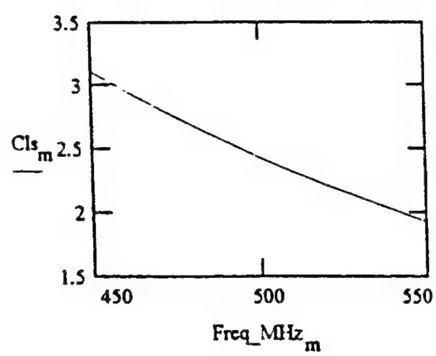
$$Llp_m = \frac{Zlp_m}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6} \cdot 10^9$$

$$Cls_m = \frac{1}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6 \cdot Zls_m} \cdot 10^{12}$$

Loop-Matching Network
Parallel Inductance vs. Frequency



Loop-Matching Network
Series Capacitance vs. Frequency



	Freq_MHz	Lds	Cdp	Cls	Llp
	1	1	1	1	1
1	450	505.85268	0.05369	3.10213	8.27578
2	460	483.85428	0.05386	2.94731	8.44848
3	470	463.22143	0.05404	2.80225	8.62545
4	480	443.87617	0.05423	2.6666	8.80504
5	490	425.70801	0.05441	2.53949	8.98773
6	500	408.61049	0.05459	2.42014	9.17388
7	510	392.51047	0.0548	2.3078	9.36457
8	520	377.33279	0.05499	2.20234	9.55756
9	530	363.00674	0.05519	2.10291	9.75465
10	540	349.46726	0.0554	2.00914	9.95575
11	550	336.65439	0.05561	1.92062	10.16064

Calculate the wave emanating from the dipole-matching network (negative active resistance)

$$\Gamma adipole_m = \left[\frac{(Za_{m,1} + j \cdot Za_{m,2}) - Z0}{(Za_{m,1} + j \cdot Za_{m,2}) + Z0}\right]$$

$$bAl_m = \frac{S12dipolemag_m \cdot \exp\left(j \cdot S12dipoleang_m \cdot \frac{\pi}{180}\right) \cdot (\Gamma adipole_m) \cdot Iin_{m,1} \cdot Z0}{[1 - (\Gamma adipole_m)]}$$

Values for 500 MHz

$Freq\ MHz_6 = 500$

$S12dipolemag_6 = 0.04422$

$S12dipoleang_6 = -88.04557$

$Za_{6,1} = -84.04509$

$Za_{6,2} = 2.9289 \cdot 10^3$

$\Gamma adipole_6 = 1.004 - 0.03414i$

$Iin_{6,1} = -i$

$bAl_6 = 0.75346 + 64.82042i$

$bAldB_6 = 10 \cdot \log\left[(|bAl_6|)^2\right]$

$bAlB_6 = 36.23482$

$|\Gamma adipole_6| = 1.00098$

$\arg(\Gamma adipole_6) \cdot \frac{180}{\pi} = -1.95443$

$|bAl_6| = 64.8248$

$\arg(bAl_6) \cdot \frac{180}{\pi} = 89.33403$

Calculate the wave incident upon the loop-matching network (positive active resistance)

$$\Gamma aloop_m = \left[\frac{(Za_{m,2} + j \cdot Za_{m,4}) - Z0}{(Za_{m,3} + j \cdot Za_{m,4}) + Z0}\right]$$

$$aBl_m = \frac{\left[1 - \left(\left|\Gamma aloop_m\right|\right)^2\right] \cdot Iin_{m,2} \cdot Z0}{S12loopmag_m \cdot \exp\left(j \cdot S12loop_m \cdot \frac{\pi}{180}\right) \cdot \left[1 - \left(a\Gamma loop_m\right)\right]}$$

Values for 500 MHz

$Freq\ MHz_6 = 500$

$S12loopmag_6 = 0.22959$

$S12loopang_6 = 144.90218$

$Za_{6,3} = 3.18291$

$Za_{6,4} = 96.16667$

$\Gamma aloop_6 = 0.55961 + 0.79632i$

$Iin_{6,2} = 5.09296$

$aBl_6 = -57.76716 - 28.12295i$

$aBldB_6 = 10 \cdot \log\left[\left(\left|aBl_6\right|\right)^2\right]$

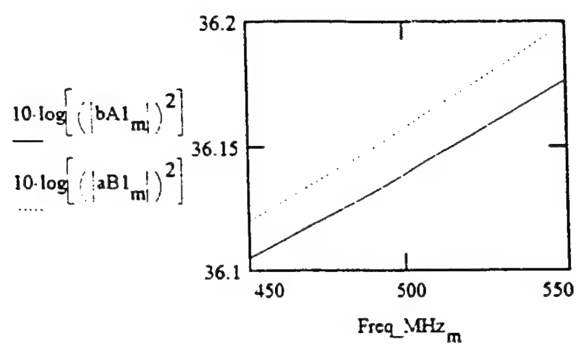
$aBlB_6 = 36.15734$

$\left|\Gamma aLOOP_6\right| = 0.97329$

$\arg(\Gamma aLOOP_6) \cdot \frac{180}{\pi} = 54.90218$

$\left|aBl_6\right| = 64.24909$

$\arg(aBl_6) \cdot \frac{180}{\pi} = 154.04166$



$$Z500 = \begin{pmatrix} 0.00934 - 2.9289 \cdot 10^{3} i & 9.13542 \cdot 10^{-5} + 16.50405 i \\ -8.52033 \cdot 10^{-5} + 15.78289 i & 0.01314 + 96.16672 i \end{pmatrix}$$

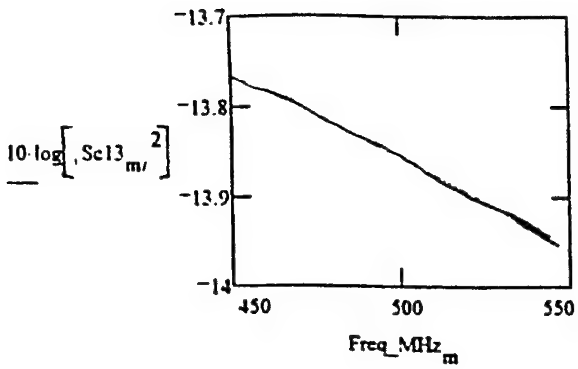
$$Y500 = \begin{pmatrix} 1.1293 \cdot 10^{-9} + 3.4111 \cdot 10^{-4} i & 7.4836 \cdot 10^{-9} - 5.8541 \cdot 10^{-5} i \\ 7.7687 \cdot 10^{-9} - 5.5983 \cdot 10^{-5} i & 1.4187 \cdot 10^{-6} - 0.01039 i \end{pmatrix}$$

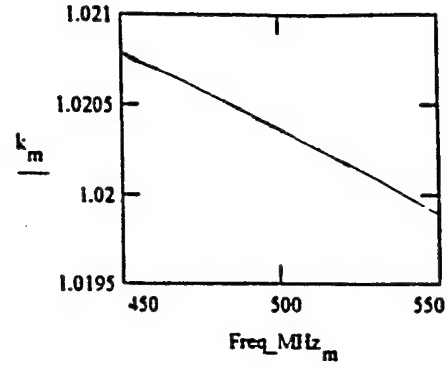
Calculate coupling parameter

$$Sc13_m = \sqrt{1 - \frac{\left(\left|bAl_m\right|\right)^2}{\left(\left|aBl_m\right|\right)^2}} \qquad \theta c13_m = \arg\left(aBl_m\right) \cdot \frac{180}{\pi} - \arg\left(bAl_m\right) \cdot \frac{180}{\pi} - 90$$

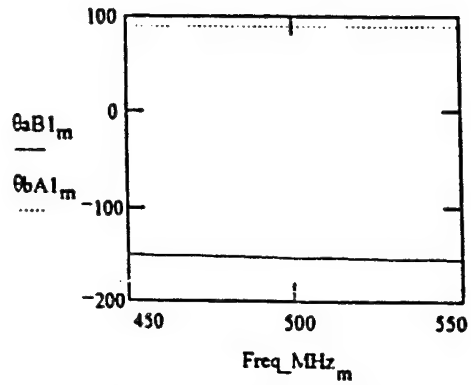
$$Sc13_6 = 0.06514i \quad Sc13dB = 20 \cdot \log\left(Sc13_6\right) \quad Sc13dB = -23.72358$$

$$k_m = \frac{\left|bAl_m\right|}{\left|aBl_m\right|} \quad k_6 = 0.99788 \quad \theta aBl_m = \arg\left(aBl_m \cdot \frac{180}{\pi}\right) \quad \theta bAl_m = \arg\left(bAl_m \cdot \frac{180}{\pi}\right)$$

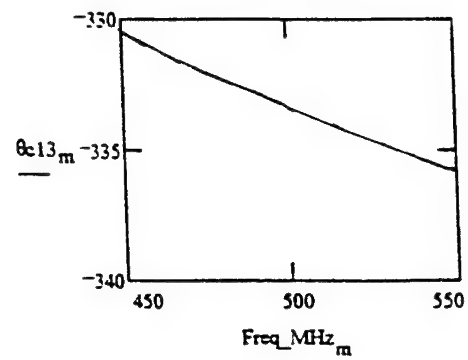




$$\frac{|bAl_6|}{|aBl_6|} = 1.02037 \arg\left(\frac{bAl_6}{aBl_6}\right)\cdot\frac{180}{\pi} = -116.65838$$



$\theta aB1_m$	$\theta bA1_m$	$\theta c13_m$
-151.08092	89.4125	-330.49342
-151.69905	89.39717	-331.09622
-152.30197	89.38154	-331.68351
-152.8867	89.36585	-332.25256
-153.45472	89.35004	-332.80476
-154.00758	89.33403	-333.34162
-154.54713	89.31772	-333.86485
-155.07011	89.3014	-334.3715
-155.58048	89.28484	-334.86532
-156.07787	89.26806	-335.34594
-156.56271	89.25112	-335.81382

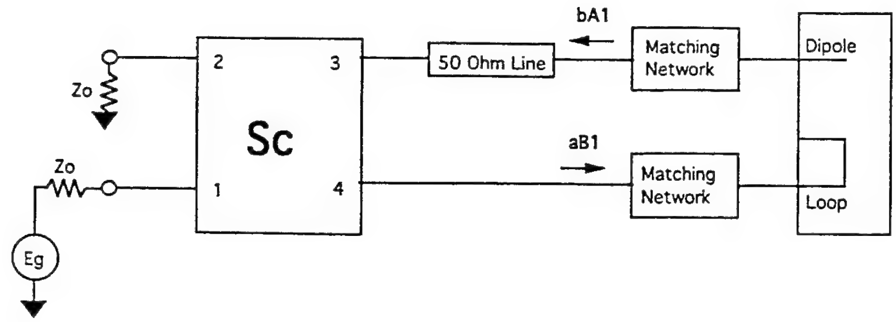


Appendix D

MATHCAD ANALYSIS FOR MIXED-MODE ARRAY (Y-RECIPROCITY NOT ENFORCED)

Consider the following mixed-mode antenna with feedback matching

Antenna Y-matrix reciprocity enforced



Read in the Y-matrix for the mixed-mode antenna. Port 1 is the dipole and port 2 is the loop.

$Y2P = \mathrm{READPRN(mixmodel)}$ $\quad\quad m = 1..11$

$j = 1..2$ $\quad\quad \mathrm{Freq}_m = 450 \cdot 10^6 + (m-1) \cdot 10 \cdot 10^6$

$k = 1..2$

$Y450_{j,k} = Y2P_{1,j \cdot 4+k \cdot 2-5} + Y2P_{1,j \cdot 4+k \cdot 2-4} \cdot i \quad Z450 = Y450^{-1}$

$Y460_{j,k} = Y2P_{2,j \cdot 4+k \cdot 2-5} + Y2P_{2,j \cdot 4+k \cdot 2-4} \cdot i \quad Z460 = Y460^{-1}$

$Y470_{j,k} = Y2P_{3,j \cdot 4+k \cdot 2-5} + Y2P_{3,j \cdot 4+k \cdot 2-4} \cdot i \quad Z470 = Y470^{-1}$

$Y480_{j,k} = Y2P_{4,j \cdot 4+k \cdot 2-5} + Y2P_{4,j \cdot 4+k \cdot 2-4} \cdot i \quad Z480 = Y480^{-1}$

$Y490_{j,k} = Y2P_{5,j \cdot 4+k \cdot 2-5} + Y2P_{5,j \cdot 4+k \cdot 2-4} \cdot i \quad Z490 = Y490^{-1}$

$Y500_{j,k} = Y2P_{6,j \cdot 4+k \cdot 2-5} + Y2P_{6,j \cdot 4+k \cdot 2-4} \cdot i \quad Z500 = Y500^{-1}$

$Y510_{j,k} = Y2P_{7,j \cdot 4+k \cdot 2-5} + Y2P_{7,j \cdot 4+k \cdot 2-4} \cdot i \quad Z510 = Y510^{-1}$

$$Y520_{j,k} = Y2P_{8,j\cdot 4+k\cdot 2-5} + Y2P_{8,j\cdot 4+k\cdot 2-4} \cdot i \quad Z520 = Y520^{-1}$$

$$Y530_{j,k} = Y2P_{9,j\cdot 4+k\cdot 2-5} + Y2P_{9,j\cdot 4+k\cdot 2-4} \cdot i \quad Z530 = Y530^{-1}$$

$$Y540_{j,k} = Y2P_{10,j\cdot 4+k\cdot 2-5} + Y2P_{10,j\cdot 4+k\cdot 2-4} \cdot i \quad Z540 = Y540^{-1}$$

$$Y550_{j,k} = Y2P_{11,j\cdot 4+k\cdot 2-5} + Y2P_{11,j\cdot 4+k\cdot 2-4} \cdot i \quad Z550 = Y550^{-1}$$

DESCRIPTION OF MIXED-MODE ARRAY

Mixed-mode antenna is comprised of a center-fed dipole (along the z-axis) whose length is 0.20 λ and whose wire is 0.001 λ. Surrounding the dipole (in the y-z plane) is a square loop with side length of 0.025 l and wire radius of 0.001 λ. The loop is fed at the point where x = 0, y = 0.0125 λ, and z = 0. This mixed-mode antenna is modelled using NEC 3D. The dipole is modelled using five segments and the loop is modelled using five segments per side. The extended kernal is used. Lossless conductors are assumed.

Compute wavelength in inches

$$\lambda inch_m = \frac{2.997925 \cdot 10^{10}}{2.54 \cdot Freq_m}$$

Enter physical length of dipole (inches)

Ldipole = 0.472114173

Enter side length of square loop (inches)

Lloop = 0.590142717

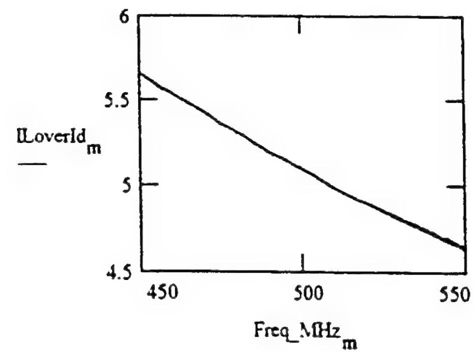
Normal to wavelength

$$Ld_m = \frac{Ldipole}{\lambda inch_m} \qquad Freq\_MHz_m = \frac{Freq_m}{10^6}$$

$$Sloop_m = \frac{Lloop}{\lambda inch_m}$$

$$ILoverId_m = \frac{Ld_m}{2 \cdot \pi \cdot (Sloop_m)^2}$$

Required Current Ratio as Function of Frequency



Dipole current

$$Iin_{m.1} = 1 \cdot \exp\left(-j \cdot 90 \frac{\pi}{180}\right)$$

$$Iin_{1.1} = -i$$

Loop current

$$Iin_{m.2} = ILoverId_m$$

$$Iin_{1,2} = 5.65884$$

Calculate the active impedance

$$j = 1..2$$

$$Z450a_j = Z450_{j,1} \cdot \frac{Iin_{1,1}}{Iin_{1,j}} + Z450_{j,2} \cdot \frac{Iin_{1,2}}{Iin_{1,j}} \quad Z450a = \begin{pmatrix} -81.57129 - 3.2564 \cdot 10^3 i \\ 2.55601 + 85.97989i \end{pmatrix}$$

$$Z460a_j = Z460_{j,1} \cdot \frac{Iin_{2,1}}{Iin_{2,j}} + Z460_{j,2} \cdot \frac{Iin_{2,2}}{Iin_{2,j}} \quad Z460a = \begin{pmatrix} -81.69161 - 3.18528 \cdot 10^3 i \\ 2.67524 + 87.99855i \end{pmatrix}$$

$$Z470a_j = Z470_{j,1} \cdot \frac{Iin_{3,1}}{Iin_{3,j}} + Z470_{j,2} \cdot \frac{Iin_{3,2}}{Iin_{3,j}} \quad Z470a = \begin{pmatrix} -81.82181 - 3.11711 \cdot 10^3 i \\ 2.79773 + 90.03315i \end{pmatrix}$$

$$Z480a_j = Z480_{j,1} \cdot \frac{Iin_{4,1}}{Iin_{4,j}} + Z480_{j,2} \cdot \frac{Iin_{4,2}}{Iin_{4,j}} \quad Z480a = \begin{pmatrix} -81.94926 - 3.05178 \cdot 10^3 i \\ 2.92309 + 92.07071i \end{pmatrix}$$

$$Z490a_j = Z490_{j,1} \cdot \frac{Iin_{5,1}}{Iin_{5,j}} + Z490_{j,2} \cdot \frac{Iin_{5,2}}{Iin_{5,j}} \quad Z490a = \begin{pmatrix} -82.07796 - 2.98912 \cdot 10^3 i \\ 3.05146 + 94.11397i \end{pmatrix}$$

$$Z500a_j = Z500_{j,1} \cdot \frac{Iin_{6,1}}{Iin_{6,j}} + Z500_{j,2} \cdot \frac{Iin_{6,2}}{Iin_{6,j}} \quad Z500a = \begin{pmatrix} -82.20864 - 2.9289 \cdot 10^3 i \\ 3.18291 + 96.16667i \end{pmatrix}$$

$$Z510a_j = Z510_{j,1} \cdot \frac{Iin_{7,1}}{Iin7_{2,j}} + Z510_{j,2} \cdot \frac{Iin_{7,2}}{Iin_{7,j}} \quad Z510a = \begin{pmatrix} -82.35137 - 2.87112 \cdot 10^3 i \\ 3.31785 + 98.23365i \end{pmatrix}$$

$$Z520a_j = Z520_{j,1} \cdot \frac{Iin_{87,1}}{Iin_{8,j}} + Z520_{j,2} \cdot \frac{Iin_{8,2}}{Iin_{8,j}} \quad Z520a = \begin{pmatrix} -84.48841 - 2.8155 \cdot 10^3 i \\ 3.45562 + 100.29979i \end{pmatrix}$$

$$Z530a_j = Z530_{j,1} \cdot \frac{Iin_{9,1}}{Iin_{9,j}} + Z530_{j,2} \cdot \frac{Iin_{9,2}}{Iin_{9,j}} \quad Z530a = \begin{pmatrix} -82.62966 - 2.76199 \cdot 10^3 i \\ 3.59664 + 102.3792i \end{pmatrix}$$

$$Z540a_j = Z540_{j,1} \cdot \frac{Iin_{10,1}}{Iin_{10,j}} + Z540_{j,2} \cdot \frac{Iin_{10,2}}{Iin_{10,j}} \quad Z540a = \begin{pmatrix} -82.77717 - 2.71046 \cdot 10^3 i \\ 3.74104 + 104.468i \end{pmatrix}$$

$$Z550a_j = Z550_{j,1} \cdot \frac{Iin_{11,1}}{Iin_{11,j}} + Z550_{j,2} \cdot \frac{Iin_{11,2}}{Iin_{11,j}} \quad Z550a = \begin{pmatrix} -82.92604 - 2.66074 \cdot 10^3 i \\ 3.88863 + 106.56654i \end{pmatrix}$$

Put active impedances into array

$$Za = \begin{array}{c} \qquad\quad \text{Dipole} \qquad\qquad\qquad\qquad\qquad \text{Loop} \\ \begin{bmatrix} \mathrm{Re}(Z450a_1) & \mathrm{Im}(Z450a_1) & \mathrm{Re}(Z450a_2) & \mathrm{Im}(Z450a_2) \\ \mathrm{Re}(Z460a_1) & \mathrm{Im}(Z460a_1) & \mathrm{Re}(Z460a_2) & \mathrm{Im}(Z460a_2) \\ \mathrm{Re}(Z470a_1) & \mathrm{Im}(Z470a_1) & \mathrm{Re}(Z470a_2) & \mathrm{Im}(Z470a_2) \\ \mathrm{Re}(Z480a_1) & \mathrm{Im}(Z480a_1) & \mathrm{Re}(Z480a_2) & \mathrm{Im}(Z480a_2) \\ \mathrm{Re}(Z490a_1) & \mathrm{Im}(Z490a_1) & \mathrm{Re}(Z490a_2) & \mathrm{Im}(Z490a_2) \\ \mathrm{Re}(Z500a_1) & \mathrm{Im}(Z500a_1) & \mathrm{Re}(Z500a_2) & \mathrm{Im}(Z500a_2) \\ \mathrm{Re}(Z510a_1) & \mathrm{Im}(Z510a_1) & \mathrm{Re}(Z510a_2) & \mathrm{Im}(Z510a_2) \\ \mathrm{Re}(Z520a_1) & \mathrm{Im}(Z520a_1) & \mathrm{Re}(Z520a_2) & \mathrm{Im}(Z520a_2) \\ \mathrm{Re}(Z530a_1) & \mathrm{Im}(Z530a_1) & \mathrm{Re}(Z530a_2) & \mathrm{Im}(Z530a_2) \\ \mathrm{Re}(Z540a_1) & \mathrm{Im}(Z540a_1) & \mathrm{Re}(Z540a_2) & \mathrm{Im}(Z540a_2) \\ \mathrm{Re}(Z550a_1) & \mathrm{Im}(Z550a_1) & \mathrm{Re}(Z550a_2) & \mathrm{Im}(Z550a_2) \end{bmatrix} \end{array}$$

Active impendances at 500 MHz

$$Z500a = \begin{pmatrix} -82.20864 - 2.9289 \cdot 10^3 i \\ 3.18291 + 96.16667i \end{pmatrix} \begin{array}{l} \mathit{Dipole} \\ \mathit{Loop} \end{array}$$

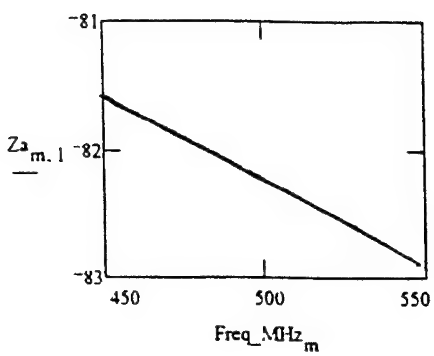
Z-matrix

$$Z500 = \begin{pmatrix} 0.00934 - 2.9289 \cdot 10^{3} i & 3.05869 \cdot 10^{-6} + 16.14346i \\ 3.05869 \cdot 10^{-6} + 16.14346i & 0.01314 + 96.16667i \end{pmatrix}$$

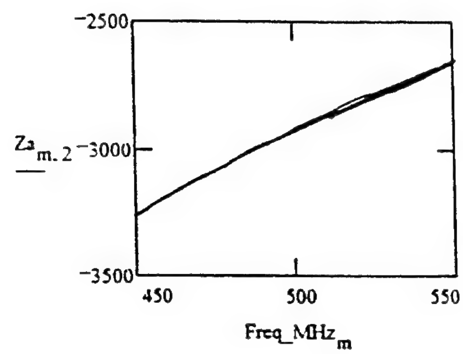
Y-matrix

$$Y500 = \begin{pmatrix} 1.1293 \cdot 10^{-9} + 3.4111 \cdot 10^{-4} i & 7.6262 \cdot 10^{-9} - 5.7262 \cdot 10^{-5} i \\ 7.6262 \cdot 10^{-9} - 5.7262 \cdot 10^{-5} i & 1.4187 \cdot 10^{-6} - 0.01039i \end{pmatrix}$$

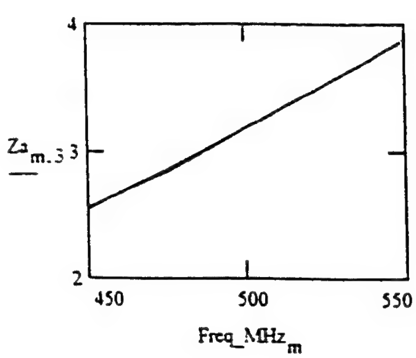
Dipole Resistance



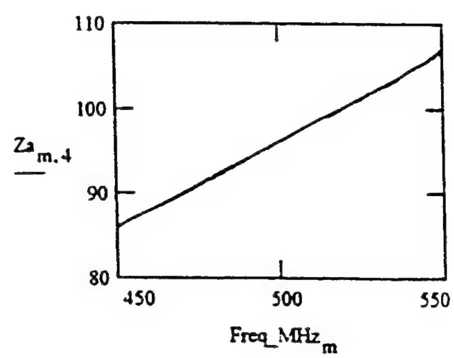
Dipole Reactance



Loop Resistance



Loop Reactance



Next, calculate the scattering parameters of the required matching networks

$$Z0 = 50$$

Matching network for positive R (S22 = Γi*)

$$S22loop_m = \overline{\left[\mathrm{Re}\left[\frac{\left(Za_{m,3} - Z0\right) + Za_{m,4} \cdot i}{\left(Za_{m,3} - Z0\right) + Za_{m,4} \cdot i}\right] + i \cdot \mathrm{Im}\left[\frac{\left(Za_{m,3} - Z0\right) + Za_{m,4} \cdot i}{\left(Za_{m,3} - Z0\right) + Za_{m,4} \cdot i}\right]\right]}$$

$$S22loopmag_m = \left|S22loop_m\right| \qquad S22loop_6 = 0.55993 - 0.79681i$$

$$S22loopang_m = \arg\left(S22loop_m\right) \cdot \frac{180}{\pi} \qquad S22loopmag_6 = 0.97387$$

$$S22loopang_6 = 54.90393$$

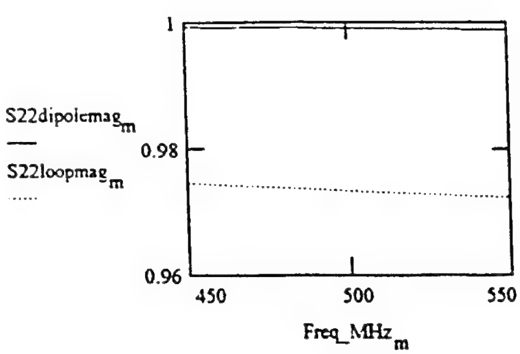
Matching network for negative R (S22 = 1/Γi)

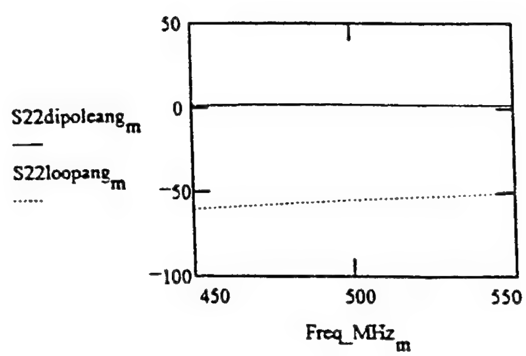
$$S22dipole_m = \mathrm{Re}\left(\frac{Za_{m,1} + Z0 + Za_{m,2} \cdot i}{Za_{m,1} - Z0 + Za_{m,2} \cdot i}\right) + i \cdot \mathrm{Im}\left(\frac{Za_{m,1} + Z0 + Za_{m,2} \cdot i}{Za_{m,1} - Z0 + Za_{m,2} \cdot i}\right)$$

$$S22dipolemag_m = \left|S22dipole_m\right| \qquad S22dipole_6 = 0.99844 + 0.03407ii$$

$$S22dipoleang_m = \arg\left(S22dipole_m\right) \cdot \frac{180}{\pi} \qquad S22dipolemag_6 = 0.99902$$

$$S22dipoleang_6 = 1.95443$$





Calculate S12 for matching network of both dipole and loop

$$S12dipolemag_m = \sqrt{1-(S22dipolemag_m)2}$$

$$S12loopmag_m = \sqrt{1-(S22loopmag_m)2}$$

$$S12dipoleang_m = S22dipoleang_m - 90$$

$$S12loopang_m = S22loopang_m - 90$$

Formulate matching network S-matrix for dipole

$$Sd450_{1,1}\,S22dipolemag_1 \cdot e^{(j\cdot S22dipoleang_1)\cdot\frac{\pi}{180}} \qquad Sd450_{2,2}\,S22dipolemag_1 \cdot e^{(j\cdot S22dipoleang_1)\cdot\frac{\pi}{180}}$$

$$Sd450_{1,2}\,S12dipolemag_1 \cdot e^{(j\cdot S12dipoleang_1)\cdot\frac{\pi}{180}} \qquad Sd450_{2,1}\,S12dipolemag_1 \cdot e^{(j\cdot S12dipoleang_1)\cdot\frac{\pi}{180}}$$

$$Sd460_{1,1}\,S22dipolemag_2 \cdot e^{(j\cdot S22dipoleang_2)\cdot\frac{\pi}{180}} \qquad Sd460_{2,2}\,S22dipolemag_2 \cdot e^{(j\cdot S22dipoleang_2)\cdot\frac{\pi}{180}}$$

$$Sd460_{1,2}\,S22dipolemag_2 \cdot e^{(j\cdot S12dipoleang_2)\cdot\frac{\pi}{180}} \qquad Sd460_{2,1}\,S12dipolemag_2 \cdot e^{(j\cdot S12dipoleang_2)\cdot\frac{\pi}{180}}$$

$$Sd470_{1,1}\,S22dipolemag_3 \cdot e^{(j\cdot S22dipoleang_3)\cdot\frac{\pi}{180}} \qquad Sd470_{2,2}\,S22dipolemag_3 \cdot e^{(j\cdot S22dipoleang_3)\cdot\frac{\pi}{180}}$$

$$Sd470_{1,2}\,S12dipolemag_3 \cdot e^{(j\cdot S12dipoleang_3)\cdot\frac{\pi}{180}} \qquad Sd470_{2,1}\,S12dipolemag_3 \cdot e^{(j\cdot S12dipoleang_3)\cdot\frac{\pi}{180}}$$

$$Sd480_{1,1}\,S22dipolemag_4 \cdot e^{(j\cdot S22dipoleang_4)\cdot\frac{\pi}{180}} \qquad Sd480_{2,2}\,S22dipolemag_4 \cdot e^{(j\cdot S22dipoleang_4)\cdot\frac{\pi}{180}}$$

$$Sd480_{1,2}\,S12dipolemag_4 \cdot e^{(j\cdot S12dipoleang_4)\cdot\frac{\pi}{180}} \qquad Sd480_{2,1}\,S12dipolemag_4 \cdot e^{(j\cdot S12dipoleang_4)\cdot\frac{\pi}{180}}$$

$$Sd490_{1,1}\,S22dipolemag_5 \cdot e^{(j\cdot S22dipoleang_5)\cdot\frac{\pi}{180}} \qquad Sd490_{2,2}\,S22dipolemag_5 \cdot e^{(j\cdot S22dipoleang_5)\cdot\frac{\pi}{180}}$$

$$Sd490_{1,2}\,S12dipolemag_5 \cdot e^{(j\cdot S12dipoleang_5)\cdot\frac{\pi}{180}} \qquad Sd490_{2,1}\,S12dipolemag_5 \cdot e^{(j\cdot S12dipoleang_5)\cdot\frac{\pi}{180}}$$

$$Sd500_{1,1}\,S22dipolemag_6 \cdot e^{(j\cdot S22dipoleang_6)\cdot\frac{\pi}{180}} \qquad Sd500_{2,2}\,S22dipolemag_6 \cdot e^{(j\cdot S22dipoleang_6)\cdot\frac{\pi}{180}}$$

$$Sd500_{1,2}\,S12dipolemag_6 \cdot e^{(j\cdot S12dipoleang_6)\cdot\frac{\pi}{180}} \qquad Sd500_{2,1}\,S12dipolemag_6 \cdot e^{(j\cdot S12dipoleang_6)\cdot\frac{\pi}{180}}$$

$Sd510_{1,1} S22dipolemag_7 \cdot e^{(j \cdot S22dipoleang_7) \cdot \frac{\pi}{180}}$ $Sd510_{2,2} S22dipolemag_7 \cdot e^{(j \cdot S22dipoleang_7) \cdot \frac{\pi}{180}}$

$Sd510_{1,2} S12dipolemag_7 \cdot e^{(j \cdot S12dipoleang_7) \cdot \frac{\pi}{180}}$ $Sd510_{2,1} S12dipolemag_7 \cdot e^{(j \cdot S12dipoleang_7) \cdot \frac{\pi}{180}}$

$Sd520_{1,1} S22dipolemag_8 \cdot e^{(j \cdot S22dipoleang_8) \cdot \frac{\pi}{180}}$ $Sd520_{2,2} S22dipolemag_8 \cdot e^{(j \cdot S22dipoleang_8) \cdot \frac{\pi}{180}}$

$Sd520_{1,2} S12dipolemag_8 \cdot e^{(j \cdot S12dipoleang_8) \cdot \frac{\pi}{180}}$ $Sd520_{2,1} S12dipolemag_8 \cdot e^{(j \cdot S12dipoleang_8) \cdot \frac{\pi}{180}}$

$Sd530_{1,1} S22dipolemag_9 \cdot e^{(j \cdot S22dipoleang_9) \cdot \frac{\pi}{180}}$ $Sd530_{2,2} S22dipolemag_9 \cdot e^{(j \cdot S22dipoleang_9) \cdot \frac{\pi}{180}}$

$Sd530_{1,2} S12dipolemag_9 \cdot e^{(j \cdot S12dipoleang_9) \cdot \frac{\pi}{180}}$ $Sd530_{2,1} S12dipolemag_9 \cdot e^{(j \cdot S12dipoleang_9) \cdot \frac{\pi}{180}}$

$Sd540_{1,1} S22dipolemag_{10} \cdot e^{(j \cdot S22dipoleang_{10}) \cdot \frac{\pi}{180}}$ $Sd540_{2,2} S22dipolemag_{10} \cdot e^{(j \cdot S22dipoleang_{10}) \cdot \frac{\pi}{180}}$

$Sd540_{1,2} S12dipolemag_{10} \cdot e^{(j \cdot S12dipoleang_{10}) \cdot \frac{\pi}{180}}$ $Sd540_{2,1} S12dipolemag_{10} \cdot e^{(j \cdot S12dipoleang_{10}) \cdot \frac{\pi}{180}}$

$Sd550_{1,1} S22dipolemag_{11} \cdot e^{(j \cdot S22dipoleang_{11}) \cdot \frac{\pi}{180}}$ $Sd550_{2,2} S22dipolemag_{11} \cdot e^{(j \cdot S22dipoleang_{11}) \cdot \frac{\pi}{180}}$

$Sd550_{1,2} S12dipolemag_{11} \cdot e^{(j \cdot S12dipoleang_{11}) \cdot \frac{\pi}{180}}$ $Sd550_{2,1} S12dipolemag_{11} \cdot e^{(j \cdot S12dipoleang_{11}) \cdot \frac{\pi}{180}}$

Formulate matching network S-matrix for loop

$Sl450_{1,1} S22loopmag_1 \cdot e^{(j \cdot S22loopang_1) \cdot \frac{\pi}{180}}$ $Sl450_{2,2} S22loopmag_1 \cdot e^{(j \cdot S22loopang_1) \cdot \frac{\pi}{180}}$

$Sl450_{1,2} S12loopmag_1 \cdot e^{(j \cdot S12loopang_1) \cdot \frac{\pi}{180}}$ $Sl450_{2,1} S12loopmag_1 \cdot e^{(j \cdot S12loopang_1) \cdot \frac{\pi}{180}}$

$Sl460_{1,1} S22loopmag_2 \cdot e^{(j \cdot S22loopang_2) \cdot \frac{\pi}{180}}$ $Sl460_{2,2} S22loopmag_2 \cdot e^{(j \cdot S22loopang_2) \cdot \frac{\pi}{180}}$

$Sl460_{1,2} S22loopmag_2 \cdot e^{(j \cdot S12loopang_2) \cdot \frac{\pi}{180}}$ $Sl460_{2,1} S12loopmag_2 \cdot e^{(j \cdot S12loopang_2) \cdot \frac{\pi}{180}}$

$$Sl470_{1,1}\,S22loopmag_3\cdot e^{(j\cdot S22loopang_3)\cdot\frac{\pi}{180}} \qquad Sl470_{2,2}\,S22loopmag_3\cdot e^{(j\cdot S22loopang_3)\cdot\frac{\pi}{180}}$$

$$Sl470_{1,2}\,S12loopmag_3\cdot e^{(j\cdot S12loopang_3)\cdot\frac{\pi}{180}} \qquad Sl470_{2,1}\,S12loopmag_3\cdot e^{(j\cdot S12loopang_3)\cdot\frac{\pi}{180}}$$

$$Sl480_{1,1}\,S22loopmag_4\cdot e^{(j\cdot S22loopang_4)\cdot\frac{\pi}{180}} \qquad Sl480_{2,2}\,S22loopmag_4\cdot e^{(j\cdot S22loopeang_4)\cdot\frac{\pi}{180}}$$

$$Sl480_{1,2}\,S12loopmag_4\cdot e^{(j\cdot S12loopang_4)\cdot\frac{\pi}{180}} \qquad Sl480_{2,1}\,S12loopmag_4\cdot e^{(j\cdot S12loopang_4)\cdot\frac{\pi}{180}}$$

$$Sl490_{1,1}\,S22loopmag_5\cdot e^{(j\cdot S22loopang_5)\cdot\frac{\pi}{180}} \qquad Sl490_{2,2}\,S22loopmag_5\cdot e^{(j\cdot S22loopang_5)\cdot\frac{\pi}{180}}$$

$$Sl490_{1,2}\,S12loopmag_5\cdot e^{(j\cdot S12loopang_5)\cdot\frac{\pi}{180}} \qquad Sl490_{2,1}\,S12loopmag_5\cdot e^{(j\cdot S12loopang_5)\cdot\frac{\pi}{180}}$$

$$Sl500_{1,1}\,S22loopmag_6\cdot e^{(j\cdot S22loopang_6)\cdot\frac{\pi}{180}} \qquad Sl500_{2,2}\,S22loopmag_6\cdot e^{(j\cdot S22loopang_6)\cdot\frac{\pi}{180}}$$

$$Sl500_{1,2}\,S12loopmag_6\cdot e^{(j\cdot S12loopang_6)\cdot\frac{\pi}{180}} \qquad Sl500_{2,1}\,S12loopmag_6\cdot e^{(j\cdot S12loopang_6)\cdot\frac{\pi}{180}}$$

$$Sl510_{1,1}\,S22loopmag_7\cdot e^{(j\cdot S22loopang_7)\cdot\frac{\pi}{180}} \qquad Sl510_{1,1}\,S22loopmag_7\cdot e^{(j\cdot S22loopang_7)\cdot\frac{\pi}{180}}$$

$$Sl510_{1,2}\,S12loopmag_7\cdot e^{(j\cdot S12loopang_7)\cdot\frac{\pi}{180}} \qquad Sl510_{2,1}\,S12loopmag_7\cdot e^{(j\cdot S12loopang_7)\cdot\frac{\pi}{180}}$$

$$Sl520_{1,1}\,S22loopmag_8\cdot e^{(j\cdot S22loopang_8)\cdot\frac{\pi}{180}} \qquad Sl520_{2,2}\,S22loopmag_8\cdot e^{(j\cdot S22loopang_8)\cdot\frac{\pi}{180}}$$

$$Sl520_{1,2}\,S12loopmag_8\cdot e^{(j\cdot S12loopang_8)\cdot\frac{\pi}{180}} \qquad Sl520_{2,1}\,S12loopmag_8\cdot e^{(j\cdot S12loopang_8)\cdot\frac{\pi}{180}}$$

$$Sl530_{1,1}\,S22loopmag_9\cdot e^{(j\cdot S22loopang_9)\cdot\frac{\pi}{180}} \qquad Sl530_{1,1}\,S22loopmag_9\cdot e^{(j\cdot S22loopang_9)\cdot\frac{\pi}{180}}$$

$$Sl530_{1,2}\,S12loopmag_9\cdot e^{(j\cdot S12loopang_9)\cdot\frac{\pi}{180}} \qquad Sl530_{1,2}\,S12loopmag_9\cdot e^{(j\cdot S12loopang_9)\cdot\frac{\pi}{180}}$$

$$Sl540_{1,1}\,S22loopmag_{10}\cdot e^{(j\cdot S22loopang_{10})\cdot\frac{\pi}{180}} \qquad Sl540_{2,2}\,S22loopmag_{10}\cdot e^{(j\cdot S22loopang_{10})\cdot\frac{\pi}{180}}$$

$$Sl540_{1,2}\,S12loopmag_{10}\cdot e^{(j\cdot S12loopang_{10})\cdot\frac{\pi}{180}} \qquad Sl540_{2,1}\,S12loopmag_{10}\cdot e^{(j\cdot S12loopang_{10})\cdot\frac{\pi}{180}}$$

$$Sl550_{1,1}\,S22loopmag_{11}\cdot e^{(j\cdot S22loopang_{11})\cdot\frac{\pi}{180}} \qquad Sl550_{2,2}\,S22loopmag_{11}\cdot e^{(j\cdot S22loopang_{11})\cdot\frac{\pi}{180}}$$

$$Sl550_{1,2}\,S12loopmag_{11}\cdot e^{(j\cdot S12loopang_{11})\cdot\frac{\pi}{180}} \qquad Sl550_{2,1}\,S12loopmag_{11}\cdot e^{(j\cdot S12loopang_{11})\cdot\frac{\pi}{180}}$$

Find the corresponding Z-matrix

$$Sd500 = \begin{pmatrix} 0.99846 + 0.03407i & 0.00149 - 0.04371i \\ 0.00149 - 0.04371i & 0.99846 + 0.03407i \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad Z0 = 50 \qquad Sl500 = \begin{pmatrix} 0.55961 - 0.79632i & -0.18785 - 0.13001i \\ -0.18785 - 0.13001i & 0.55961 - 0.79632i \end{pmatrix}$$

$$Zd450 = (I - Sd450)^{-1} \cdot (I + Sd450) \cdot Z0 \quad Zl450 = (I - Sl450)^{-1} \cdot (I + Sl450) \cdot Z0$$
$$Zd460 = (I - Sd460)^{-1} \cdot (I + Sd460) \cdot Z0 \quad Zl460 = (I - Sl460)^{-1} \cdot (I + Sl460) \cdot Z0$$
$$Zd470 = (I - Sd470)^{-1} \cdot (I + Sd470) \cdot Z0 \quad Zl470 = (I - Sl470)^{-1} \cdot (I + Sl470) \cdot Z0$$
$$Zd480 = (I - Sd480)^{-1} \cdot (I + Sd480) \cdot Z0 \quad Zl480 = (I - Sl480)^{-1} \cdot (I + Sl480) \cdot Z0$$
$$Zd490 = (I - Sd490)^{-1} \cdot (I + Sd490) \cdot Z0 \quad Zl490 = (I - Sl490)^{-1} \cdot (I + Sl490) \cdot Z0$$
$$Zd500 = (I - Sd500)^{-1} \cdot (I + Sd500) \cdot Z0 \quad Zl500 = (I - Sl500)^{-1} \cdot (I + Sl500) \cdot Z0$$
$$Zd510 = (I - Sd510)^{-1} \cdot (I + Sd510) \cdot Z0 \quad Zl510 = (I - Sl510)^{-1} \cdot (I + Sl5120) \cdot Z0$$
$$Zd520 = (I - Sd520)^{-1} \cdot (I + Sd520) \cdot Z0 \quad Zl520 = (I - Sl520)^{-1} \cdot (I + Sl520) \cdot Z0$$
$$Zd530 = (I - Sd530)^{-1} \cdot (I + Sd530) \cdot Z0 \quad Zl530 = (I - Sl530)^{-1} \cdot (I + Sl530) \cdot Z0$$
$$Zd540 = (I - Sd540)^{-1} \cdot (I + Sd540) \cdot Z0 \quad Zl540 = (I - Sl540)^{-1} \cdot (I + Sl540) \cdot Z0$$
$$Zd550 = (I - Sd550)^{-1} \cdot (I + Sd550) \cdot Z0 \quad Zl550 = (I - Sl550)^{-1} \cdot (I + Sl550) \cdot Z0$$

Next, find elements of equivalent tee-network

$$Zd500 \begin{pmatrix} -4.54675 \cdot 10^3 i & -5.83044 \cdot 10^3 i \\ -5.83044 \cdot 10^3 i & -4.54675 \cdot 10^3 i \end{pmatrix}$$

$$Zl500 \begin{pmatrix} -102.70466i & 28.8206i \\ 28.8206i & -102.70466i \end{pmatrix}$$

$$Zds_1 = Zd450_{1,1} - Zd450_{1,2} \qquad Zdp_1 = Zd450_{1,2}$$
$$Zds_2 = Zd460_{1,1} - Zd460_{1,2} \qquad Zdp_2 = Zd460_{1,2}$$
$$Zds_3 = Zd470_{1,1} - Zd470_{1,2} \qquad Zdp_3 = Zd470_{1,2}$$
$$Zds_4 = Zd480_{1,1} - Zd480_{1,2} \qquad Zdp_4 = Zd480_{1,2}$$

$Zds_5 = Zd490_{1,1} - Zd490_{1,2}$ $\quad Zdp_5 = Zd490_{1,2}$

$Zds_6 = Zd500_{1,1} - Zd500_{1,2}$ $\quad Zdp_6 = Zd500_{1,2}$

$Zds_7 = Zd510_{1,1} - Zd510_{1,2}$ $\quad Zdp_7 = Zd510_{1,2}$

$Zds_8 = Zd520_{1,1} - Zd520_{1,2}$ $\quad Zdp_8 = Zd520_{1,2}$

$Zds_9 = Zd530_{1,1} - Zd530_{1,2}$ $\quad Zdp_9 = Zd530_{1,2}$

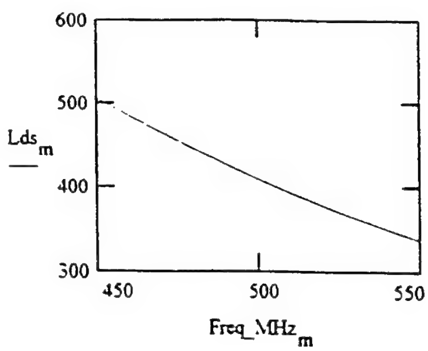
$Zds_{10} = Zd540_{1,1} - Zd540_{1,2}$ $\quad Zdp_{10} = Zd540_{1,2}$

$Zds_{11} = Zd550_{1,1} - Zd550_{1,2}$ $\quad Zdp_{11} = Zd550_{1,2}$

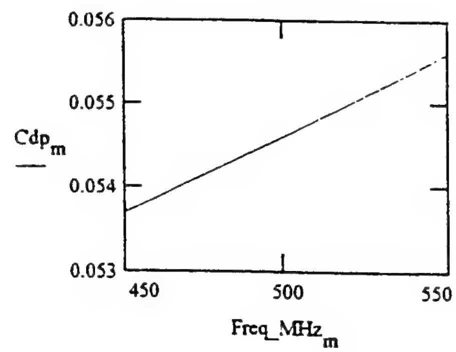
$$Lds_m = \frac{Zds_m}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6} \cdot 10^9$$

$$Cdp_m = \frac{1}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6 \cdot Zdp_m} \cdot 10^{12}$$

Dipole-Matching Network
Series Inductance vs. Frequency



Dipole-Matching Network
Parallel Capacitance vs. Frequency



$Zls_1 = Zl450_{1,1} - Zl450_{1,2}$ $\quad Zlp_1 = Zl450_{1,2}$ $\quad Zlp_1 = 23.39923i$

$Zls_2 = Zl460_{1,1} - Zl460_{1,2}$ $\quad Zlp_2 = Zl460_{1,2}$ $\quad Zlp_2 = 24.41836i$

$Zls_3 = Zl470_{1,1} - Zl470_{1,2}$ $\quad Zlp_3 = Zl470_{1,2}$

$Zls_4 = Zl480_{1,1} - Zl480_{1,2}$ $\quad Zlp_4 = Zl480_{1,2}$

$Zls5 = Zl490_{1,1} - Zl490_{1,2}$ $\quad Zlp_5 = Zl490_{1,2}$

$Zls6 = Zl500_{1,1} - Zl500_{1,2}$ $\quad Zlp6 = Zl5000_{1,2}$

$Zls_7 = Zl510_{1,1} - Zl510_{1,2}$ $\quad Zlp_7 = Zl510_{1,2}$

$Zls_8 = Zl520_{1,1} - Zl520_{1,2}$ $\quad Zlp_8 = Zl520_{1,2}$

$$Zls9 = Zl530_{1,1} - Zl530_{1,2} \qquad Zlp_9 = Zl530_{1,2}$$

$$Zls_{10} = Zl540_{1,1} - Zl540_{1,2} \qquad Zlp10 = Zl540_{1,2}$$

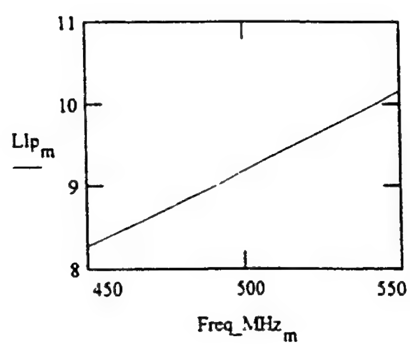
$$Zls_{11} = Zl550_{1,1} - Zl550_{1,2} \qquad Zlp_{11} = Zl550_{1,2}$$

$$Zls_6 = 131.52525i \quad Zlp10 = 33.77907i$$

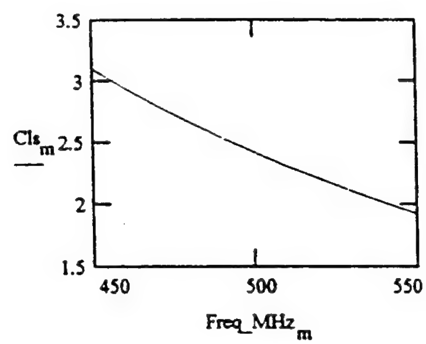
$$Llp_m = \frac{Zlp_m}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6} \cdot 10^9$$

$$Cls_m = \frac{1}{j \cdot 2 \cdot \pi \cdot Freq\ MHz_m \cdot 10^6 \cdot Zls_m} \cdot 10^{12}$$

Loop-Matching Network
Parallel Inductance vs. Frequency



Loop-Matching Network
Series Capacitance vs. Frequency



	Freq_MHz =	Lds =	Cdp =	Cls =	Llp =
	1	1	1	1	1
1	450	505.85268	0.05369	3.10213	8.27578
2	460	483.85428	0.05386	2.94731	8.44848
3	470	463.22143	0.05404	2.80225	8.62545
4	480	443.87617	0.05423	2.6666	8.80504
5	490	425.70801	0.05441	2.53949	8.98773
6	500	408.61049	0.05459	2.42014	9.17388
7	510	392.51047	0.0548	2.3078	9.36457
8	520	377.33279	0.05499	2.20234	9.55756
9	530	363.00674	0.05519	2.10291	9.75465
10	540	349.46726	0.0554	2.00914	9.95575
11	550	336.65439	0.05561	1.92062	10.16064

Calculate the wave emanating from the dipole-matching network (negative active resistance)

$$\Gamma adipole_m = \left[\frac{\left(Za_{m,1} + j \cdot Za_{m,2}\right) - Z0}{\left(Za_{m,1} + j \cdot Za_{m,2}\right) + Z0}\right]$$

$$bAl_m = \frac{S12dipolemag_m \cdot \exp\left(j \cdot S12dipoleang_m \cdot \frac{\pi}{180}\right) \cdot \left(\Gamma adipole_m\right) \cdot Iin_{m,1} \cdot Z0}{\left[1 - \left(\Gamma adipole_m\right)\right]}$$

Values for 500 MHz

$Freq\ MHz_6 = 500$

$S12dipolemag_6 = 0.04373$

$S12dipoleang_6 = -88.0455$

$Za_{6,1} = -82.20864$

$Za_{6,2} = 2.9289 \cdot 10^3$

$\Gamma adipole_6 = 1.00038 - 0.03414i$

$Iin_{6,1} = -i$

$bAl_6 = 0.70499 + 64.10877i$

$bAldB_6 = 10 \cdot \log\left[\left(\left|bAl_6\right|\right)^2\right]$

$bAlB_6 = 36.13887$

$\left|\Gamma adipole_6\right| = 1.00096$

$\arg\left(\Gamma adipole_6\right) \cdot \frac{180}{\pi} = -1.9545$

$\left|bAl_6\right| = 64.11265$

$\arg\left(bAl_6\right) \cdot \frac{180}{\pi} = 89.36995$

Calculate the wave incident upon the loop-matching network (positive active resistance)

$$\Gamma aloop_m = \left[\frac{\left(Za_{m,2} + j \cdot Za_{m,4}\right) - Z0}{\left(Za_{m,3} + j \cdot Za_{m,4}\right) + Z0}\right]$$

$$aBl_m = \frac{\left[1 - \left(\left|\Gamma aloop_m\right|\right)^2\right] \cdot Iin_{m,2} \cdot Z0}{S12loopmag_m \cdot \exp\left(j \cdot S12loop_m \cdot \frac{\pi}{180}\right) \cdot \left[1 - \left(a\Gamma loop_m\right)\right]}$$

Values for 500 MHz

$Freq\ MHz_6 = 500$

$S12loopmag_6 = 0.22959$

$S12loopang_6 = 144.90218$

$Za_{6,3} = 3.18291$

$Za_{6,4} = 96.16667$

$\Gamma aloop_6 = 0.55961 + 0.79632i$

$Iin_{6,2} = 5.09296$

$aBl_6 = -57.76716 - 28.12295i$

$aBldB_6 = 10 \cdot \log\left[\left(\left|aBl_6\right|\right)^2\right]$

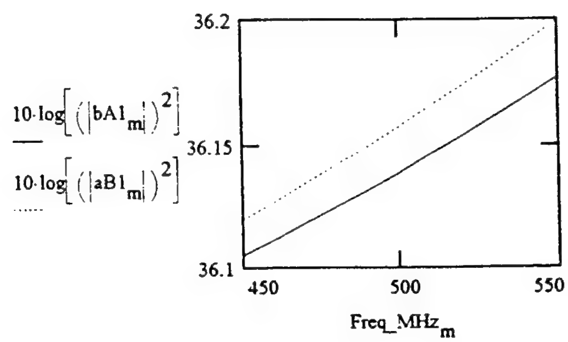
$aBlB_6 = 36.15734$

$\left|\Gamma aLOOP_6\right| = 0.97329$

$\arg\left(\Gamma aLOOP_6\right) \cdot \dfrac{180}{\pi} = 54.90218$

$\left|aBl_6\right| = 64.24909$

$\arg\left(aBl_6\right) \cdot \dfrac{180}{\pi} = 154.04166$



$$Z500 = \begin{pmatrix} 0.00934 - 2.9289 \cdot 10^{3} i & 3.05869 \cdot 10^{-6} + 16.14346i \\ 3.05869 \cdot 10^{-6} + 16.14346i & 0.01314 + 96.1667i \end{pmatrix}$$

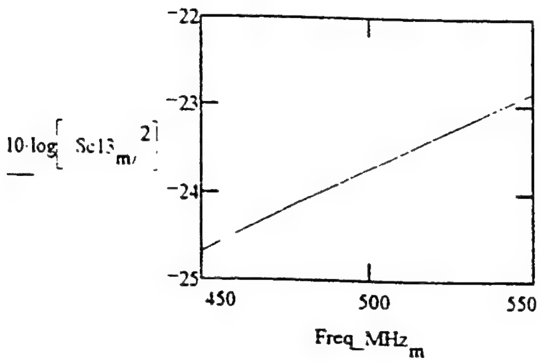
$$Y500 = \begin{pmatrix} 1.1293 \cdot 10^{-9} + 3.4111 \cdot 10^{-4} i & 7.6262 \cdot 10^{-9} - 5.7262 \cdot 10^{-5} i \\ 7.6262 \cdot 10^{-9} - 5.7262 \cdot 10^{-5} i & 1.4187 \cdot 10^{-6} - 0.01039i \end{pmatrix}$$

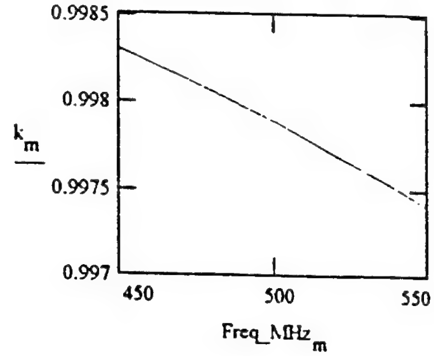
Calculate coupling parameter

$$Sc13_m = \sqrt{1 - \frac{\left(\left|bAl_m\right|\right)^2}{\left(\left|aBl_m\right|\right)^2}} \qquad \theta c13_m = \arg\left(aBl_m\right) \cdot \frac{180}{\pi} - \arg\left(bAl_m\right) \cdot \frac{180}{\pi} - 90$$

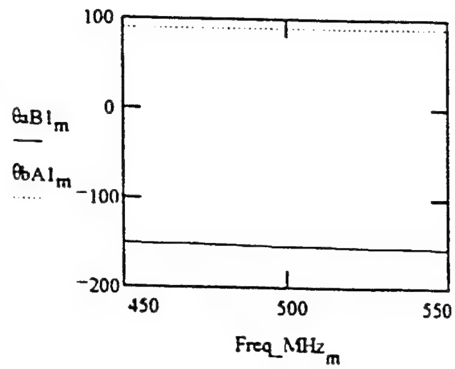
$Sc13_6 = 0.20288i \quad Sc13dB = 20 \cdot \log\left(Sc13_6\right) \quad Sc13dB = -13.85522 + 13.64376i$

$$k_m = \frac{|bAl_m|}{|aBl_m|} \quad k_6 = 1.02037 \quad \theta aBl_m = \arg\left(aBl_m \cdot \frac{180}{\pi}\right) \quad \theta bAl_m = \arg\left(bAl_m \cdot \frac{180}{\pi}\right)$$

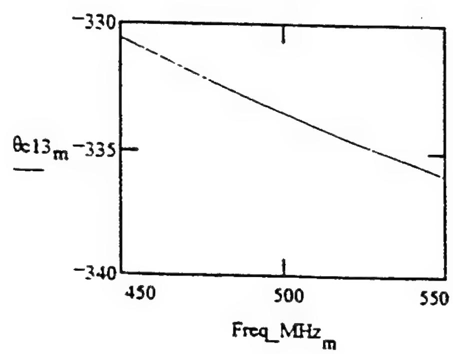




$$\frac{|bAl_6|}{|aBl_6|} = 1.02037 \arg\left(\frac{bAl_6}{aBl_6}\right) \cdot \frac{180}{\pi} = -116.65838$$



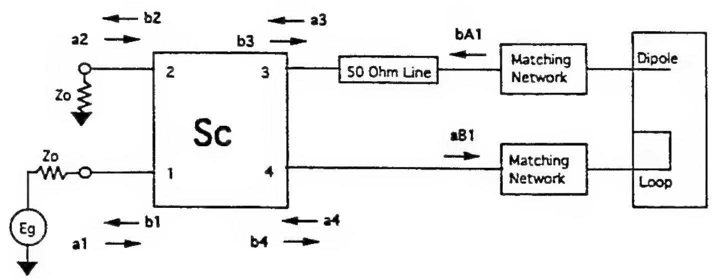
$\theta aB1_m$	$\theta bA1_m$	$\theta c13_m$
-151.10994	89.44453	-330.55446
-151.72907	89.42996	-331.15903
-152.33303	89.4151	-331.74813
-152.91877	89.40019	-332.31896
-153.48779	89.38515	-332.87294
-154.04166	89.36995	-333.41161
-154.58221	89.35443	-333.93664
-155.10622	89.33889	-334.4451
-155.61759	89.32315	-334.94074
-156.11599	89.30716	-335.42316
-156.60185	89.29102	-335.89287



Appendix E

MATHCAD ANALYSIS TO CALCULATE REQUIRED COUPLING FACTOR AND ADDITIONAL PHASE SHIFT

Mathcad program to calculate the coupling and phase shift required for mixed-mode antenna matching-network feedback



Desire b2 = 0

$$b2 = s21 \cdot al + S22 \cdot a2 + S23 \cdot a3 + S24 \cdot a4$$

Assume matched conditions such that a2 = 0 and a4 = 0

$$b2 = S21 \cdot al + S23 \cdot a3$$

Using Dave's notation and relating required phase shift line length

$$a3 = \exp(-j \cdot \theta) \cdot bAl$$

Substituting

$$b2 = S21 \cdot al + S23 \cdot \left(\exp(-j \cdot \theta) \cdot bAl\right)$$

Since we desire that b2 = 0

$$al = -S23 \cdot bAl \cdot \frac{\exp(-j \cdot \theta)}{S21}$$

Substituting for a1

$$aBl = S41\cdot\left(-S23\cdot bAl\cdot\frac{\exp(-j\cdot\theta)}{S21}\right)+S43\cdot\left(\exp(-j\cdot\theta)\cdot bAl\right)$$

$$aBl = \left(-S41\cdot S23\cdot\frac{\exp(-j\cdot\theta)}{S21}+S43\cdot\exp(-j\cdot\theta)\right)\cdot bAl$$

$$aBl = \left(-S41\cdot\frac{S23}{S21}+S43\right)\cdot bAl\cdot\exp(-j\cdot\theta)$$

Examine S-parameters of directional coupler

Directly coupled ports can be described by Sd = S14, S41, S23, S32
Cross-coupled ports can be described by Sc = S12, S21, S43, S34

Substituting Sd and Sc

$$aBl = \left(-Sd\cdot\frac{Sd}{Sc}+Sc\right)\cdot bAl\cdot\exp(-j\cdot\theta)$$

$$aBl = \left(\frac{Sc^2-Sd^2}{Sc}\right)\cdot bA1\cdot\exp(-j\cdot\theta)$$

Write expression for Sd and Sc (from IRE Trans MTT "Coupled-Strip-Transmission-Line Filters and Directional Couplers," Jones and Bolljahn, April 1956).

$$k = \frac{Zoo-Zoe}{Zoo+Zoe}$$

Writing expression for coupler S-parameters as function of k at frequency where coupling length is a quarter wavelength

$$Sd = -j\cdot\sqrt{1-k^2}\quad Sc = k\quad Sd = -0.06508\quad Sc = -0.99788$$

Substituting for b1

$$aBl = \left(\frac{Sc^2 - Sd^2}{Sc}\right) \cdot bAl \cdot \exp(-j \cdot \theta)$$

$$aBl = \left[\frac{(-k)^2 - \left(-j \cdot \sqrt{1-k^2}\right)^2}{-k}\right] \cdot bAl \cdot \exp(-j \cdot \theta)$$

$$aBl = \frac{-1}{k} \cdot bA1 \cdot \exp(-j \cdot \theta)$$

Solving for the ratio of bAl/aBl in terms of k and theta.

Define the feedback ratio (bAl/aBl)=K as

$$K = -k \cdot \exp(j \cdot \theta)$$

$$K = k \cdot \exp(j \cdot (+180))$$

$$\theta = \arg\left(\frac{bAl}{aBl}\right) - 180$$

Compute value of additional phase shift.

Enter bAl and aB1

$$bAl = 0.70499 + j \cdot 64.10877$$

$$aBl = -57.76716 - j \cdot 28.12295$$

$$\arg(bAl) \cdot \frac{180}{\pi} = 89.36996$$

$$\arg(aBl) \cdot \frac{180}{\pi} = -154.04166$$

$$\arg\left(\frac{bAl}{aBl}\right) \cdot \frac{180}{\pi} = -116.58838$$

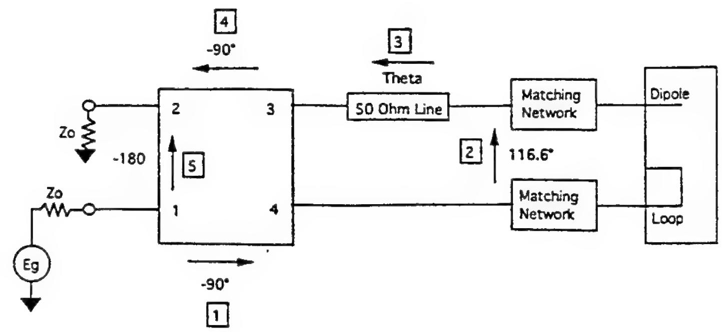
$$\theta = \arg\left(\frac{bAl}{aBl}\right) \cdot \frac{180}{\pi} - 180$$

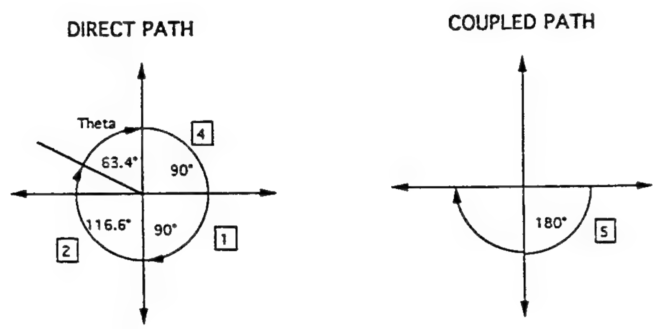
$$\theta = -296.58838$$

Or in terms of positive phase shift

$$\theta = \left(\arg\left(\frac{bAl}{aBl}\right) \cdot \frac{180}{\pi} - 180 \right) + 360$$

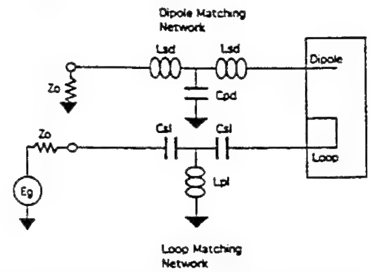
$$\theta = 63.41162$$





Appendix F

TOUCHSTONE ANALYSIS OF MIXED-MODE ARRAY (NO FEEDBACK)

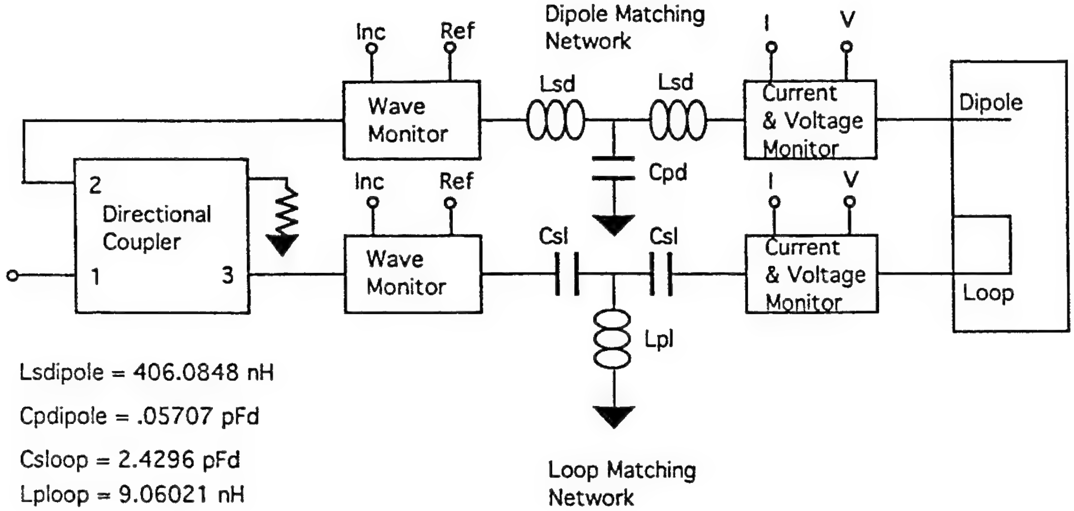



```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
              MIXMODE1.CKT    Tue Jan 31 18:00:28 1995

DIM
  FREQ GHZ
  RES OH
  IND NH
  CAP PF
  LNG MIL
  TIME PS
  COND /OH
  ANG DEG
 ***********************************************************************
VAR
 *****************************************************************
!   VARIABLES ASSOCIATED WITH MIXED MODE ANTENNA
!*****************************************************************
!********************************************************************
 DIPOLE MATCHING NETWORK (TEE SECTION)
!********************************************************************
  LSD = 406.0848
  CPD = .05707
!********************************************************************
!LOOP MATCHING NETWORK (TEE SECTION)
!********************************************************************
  CSL = 2.4296
  LPL = 9.06021
!********************************************************************
EQN
!********************************************************************
CKT
 ********************************************************************
! Antenna Y-matrix (passive) from NEC MOM
  S2PA 1 2 0 mixmode2.S2P
  DEF2P 1 2 ANTENNA
  current & voltage monitor
  S4PA 1 2 3 4 IVMETER.S4P
  DEF4P 1 2 3 4 IVMETER
  reflection coefficient monitor
  S4PB 1 2 3 4 REFLMETR.S4P
  DEF4P 1 2 3 4 REFLMON
 ********************************************************************
! Dipole Matching Network
!********************************************************************
   IND 1 2 L^LSD
   CAP 2 0 C^CPD
   IND 2 3 L^LSD
   DEF2P 1 3 DMN
 ********************************************************************
```

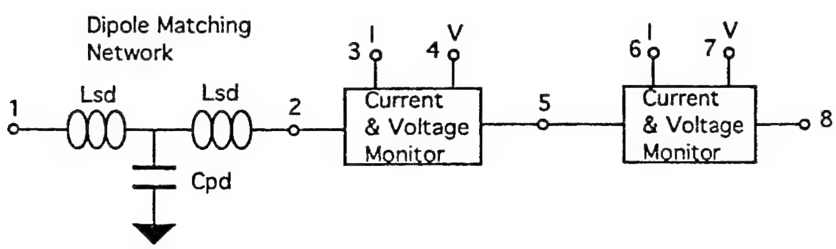
```
! Loop Matching Network
!*************************************************************************
  CAP 2 1 C^CSL
  IND 2 0 L^LPL
  CAP 2 3 C^CSL
  DEF2P 1 3 LMN
*************************************************************************
! Define 6-port Channel #1  (DIPOLE)
*************************************************************************
  REFLMON 1 4 2 3
  DMN 4 5
  IVMETER 5 8 6 7
  DEF6P 1 2 3 6 7 8 CHAN1
!*************************************************************************
  Define 6-port Channel #2  (LOOP)
!*************************************************************************
  REFLMON 1 4 2 3
  LMN 4 5
  IVMETER 5 8 6 7
  DEF6P 1 2 3 6 7 8 CHAN2
*************************************************************************
  Define 2-port to measure Passive S-Parameters of antenna and matching NW
!*************************************************************************
  DMN 1 2
  LMN 4 3
  ANTENNA 2 3
  DEF2P 1 4  SPASSV
*************************************************************************
! Define 3-port to measure Reflected waves at input to matching Networks
! Select REFL S21 to measure reflected wave in channel 1 (DIPOLE)
  Select REFL S31 to measure reflected wave in channel 2 (LOOP)
*************************************************************************
  RES 1 0 R=50
  CHAN1 1  2  3  4  5  6
  CHAN2 7  8  9 10 11 12
  ANTENNA  6 12
  DEF3P 7 3 9 REFL
*************************************************************************
! Define 3-port to measure Incident waves at input to matching Networks
! Select INC S21 to measure incident wave in channel 1 (DIPOLE)
  Select INC S31 to measure incident wave in channel 2 (LOOP)
*************************************************************************
  RES 1 0 R=50
  CHAN1 1  2  3  4  5  6
  CHAN2 7  8  9 10 11 12
  ANTENNA 6 12
  DEF3P 7 2 8 INC
*************************************************************************
! Define 3-port to measure VANT (voltage at antenna) for all Channels
! Select VANT S21 for Voltage at DIPOLE
  Select VANT S31 for Voltage at LOOP
*************************************************************************
  RES 1 0 R=50
  CHAN1 1  2  3  4  5  6
  CHAN2 7  8  9 10 11 12
  ANTENNA 6 12
  DEF3P 7 5 11 VANT
*************************************************************************
```

```
! Define 3-port to measure IANT (current AT antenna) for all Channels
! Select IANT S21 for current at DIPOLE
: Select IANT S31 for current at LOOP
;**********************************************************************
  RES 1 0 R=50
  CHAN1 1  2  3  4  5  6


 CHAN2 7  8  9 10 11 12
 ANTENNA 6 12
 DEF3P 7 4 10 IANT
!**********************************************************************
' Define 6-port Current Reference Channel (#1) DIPOLE
 *********************************************************************
  DMN 1 2
  IVMETER 2 5 3 4
  IVMETER 5 8 6 7
  DEF6P 1 3 4 6 7 8 CHANREF
!**********************************************************************
  Define 3 port current reference (Normalize antenna currents to DIPOLE)
 *********************************************************************
  RES 1 0 R=50
  CHANREF 1  2  3  4  5  6
  CHAN2   7  8  9 10 11 12
  ANTENNA 6 12
  DEF3P 7 2 4 [REF
:ERM
.'ROC
  GAMMA = REFL / INC
  ZANT  = VANT / IANT
  INORM = IANT / IREF
OUT
'Passive Antenna Data
   ANTENNA MAG[S11]
!  ANTENNA ANG[S11]
'  ANTENNA MAG[S12]
   ANTENNA ANG[S12]
;  ANTENNA MAG[S21]
!  ANTENNA ANG[S21]
   ANTENNA MAG[S22]
.  ANTENNA ANG[S22]
!Passive matching network S-paramters
  DMN MAG[S11]
  DMN ANG[S11]
! LMN MAG[S11]
' LMN ANG[S11]
  DMN MAG[S21]
: DMN ANG[S21]
! LMN MAG[S21]
  LMN ANG[S21]
. DMN MAG[S22]
! DMN ANG[S22]
  LMN MAG[S22]
  LMN ANG[S22]
 ! Passive Scattering Parameters of Antenna and Matching Networks
 ' SPASSV DB[S11] GR1
   SPASSV ANG[S11] GR2
 : SPASSV DB[S22]  GR1
```

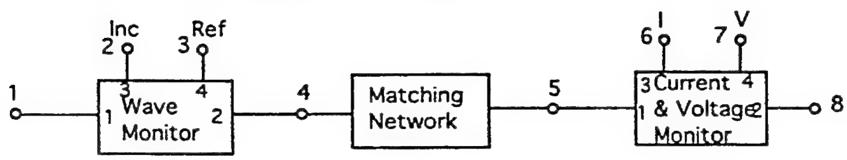
```
! SPASSV ANG[S22] GR2
  SPASSV DB[S12]  GR1
. SPASSV ANG[S12] GR2
! SPASSV DB[S21]  GR1
  SPASSV ANG[S21] GR2
!Reflection coefficients
!   GAMMA S21 SC2
!   GAMMA S31 SC2
!Incident waves
!   INC MAG[S21]
!   INC ANG[S21]
!   INC MAG[S31]
!   INC ANG[S31]
!   REFL MAG[S21]
!   REFL ANG[S21]
!   REFL MAG[S31]
!   REFL ANG[S31]
!Antenna Voltages
!   VANT S21
!   VANT S31
!Antenna Currents
!   IANT S21
!   IANT S31
!Active impedances
!  ZANT  RE[S21]
!  ZANT  IM[S21]
!  ZANT  RE[S31]
!  ZANT  IM[S31]
!Reference currents
   IREF MAG[S21]
   IREF ANG[S21]
   IREF MAG[S31]
   IREF ANG[S31]
!Normalized Current Ratios
!  Inorm MAG[S21]
!  Inorm ANG[S21]
!  Inorm MAG[S31]
!  Inorm ANG[S31]
FREQ
   SWEEP .450 .550 .010
!  STEP .500
GRID
  RANGE .450 .550 .010
  GR1 0 -60 10
  GR2 180 -180 30
OPT
```



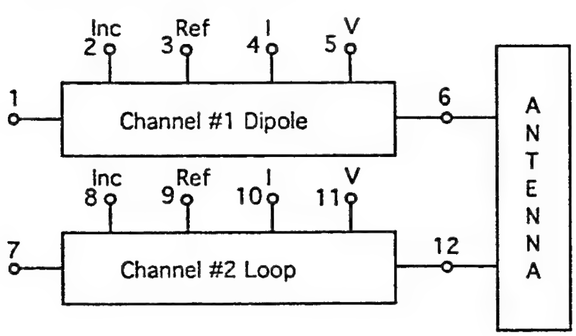
CURRENT REFERENCE CHANNEL



CHANNEL TOPOLOGY



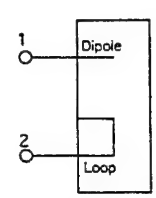
OVERALL TOPOLOGY



MIXMODE1.Y2P

```
! Simple Dipole Antenna located on Z-axis, center fed
! Dipole length = .02 wavelengths at 500 MHz
! Dipole radius = .001 wavelengths at 500 MHz
! Square loop in yz plane with side length = .025 wavelengths at 5
00 MHz
# GHZ Y RI R 1 !required
!F(GHz) Y11r     Y11i        Y21r         Y21i        Y12r        Y12i
  Y22r       Y22i
!.450 7.4255E-10 3.0686E-4 5.0949E-9 -5.0303E-5 4.9090E-9 -5.2599E
-5 1.1422E-6 -1.1622E-2
!.460 8.1044E-10 3.1370E-4 5.5636E-9 -5.1437E-5 5.3603E-9 -5.3785E
-5 1.1949E-6 -1.1355E-2
!.470 8.8286E-10 3.2055E-4 6.0639E-9 -5.2572E-5 5.8421E-9 -5.4973E
-5 1.2489E-6 -1.1098E-2
!.480 9.6001E-10 3.2740E-4 6.5972E-9 -5.3708E-5 6.3556E-9 -5.6161E
-5 1.3042E-6 -1.0852E-2
!.490 1.0421E-09 3.3425E-4 7.1650E-9 -5.4845E-5 6.9023E-9 -5.7350E
-5 1.3608E-6 -1.0616E-2
.500 1.1293E-09 3.4111E-4 7.7687E-9 -5.5983E-5 7.4836E-9 -5.8541E-
5 1.4187E-6 -1.0389E-2
!.510 1.2218E-09 3.4796E-4 8.4098E-9 -5.7123E-5 8.1008E-9 -5.9733E
-5 1.4779E-6 -1.0170E-2
!.520 1.3198E-09 3.5482E-4 9.0899E-9 -5.8264E-5 8.7554E-9 -6.0927E
-5 1.5385E-6 -9.9601E-3
!.530 1.4236E-09 3.6168E-4 9.8105E-9 -5.9405E-5 9.4490E-9 -6.2121E
-5 1.6004E-6 -9.7574E-3
!.540 1.5334E-09 3.6854E-4 1.0573E-8 -6.0549E-5 1.0183E-8 -6.3317E
-5 1.6636E-6 -9.5619E-3
!.550 1.6493E-09 3.7541E-4 1.1379E-8 -6.1693E-5 1.0959E-8 -6.4514E
-5 1.7282E-6 -9.3732E-3
```

FREQ-GHZ	MAG[S11] ANTENNA	ANG[S11] ANTENNA	MAG[S12] ANTENNA	ANG[S12] ANTENNA	MAG[S21] ANTENNA	ANG[S21] ANTENNA	MAG[S22] ANTENNA	ANG[S22] ANTENNA
0.45000	1.000	-1.758	0.004	119.286	0.005	119.286	1.000	60.322
0.46000	1.000	-1.798	0.004	118.692	0.005	118.691	1.000	59.171
0.47000	1.000	-1.837	0.005	118.113	0.005	118.112	1.000	58.052
0.48000	1.000	-1.876	0.005	117.552	0.005	117.551	1.000	56.969
0.49000	1.000	-1.915	0.005	117.008	0.005	117.007	1.000	55.919
0.50000	1.000	-1.955	0.005	116.479	0.005	116.478	1.000	54.899
0.51000	1.000	-1.994	0.005	115.963	0.005	115.962	1.000	53.907
0.52000	1.000	-2.033	0.005	115.464	0.005	115.463	1.000	52.947
0.53000	1.000	-2.072	0.005	114.978	0.006	114.977	1.000	52.013
0.54000	1.000	-2.112	0.005	114.505	0.006	114.504	1.000	51.104
0.55000	1.000	-2.151	0.006	114.044	0.006	114.043	1.000	50.221

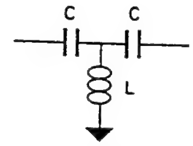


S-parameters in 50 ohm system

Y-Matrix Symmetry not enforced

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE1.OUT Tue Jan 31 14:32:15 1995

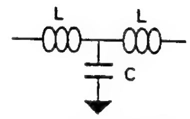
FREQ-GHZ	MAG[S11] LMN	ANG[S11] LMN	MAG[S21] LMN	ANG[S21] LMN	MAG[S22] LMN	ANG[S22] LMN
0.45000	0.988	-46.881	0.156	-136.881	0.988	-46.881
0.46000	0.986	-48.392	0.168	-138.392	0.986	-48.392
0.47000	0.983	-49.948	0.182	-139.948	0.983	-49.948
0.48000	0.981	-51.550	0.196	-141.550	0.981	-51.550
0.49000	0.977	-53.201	0.211	-143.201	0.977	-53.201
0.50000	0.974	-54.904	0.227	-144.904	0.974	-54.904
0.51000	0.970	-56.661	0.244	-146.661	0.970	-56.661
0.52000	0.965	-58.476	0.262	-148.476	0.965	-58.476
0.53000	0.960	-60.350	0.281	-150.350	0.960	-60.350
0.54000	0.954	-62.287	0.300	-152.287	0.954	-62.287
0.55000	0.947	-64.289	0.321	-154.289	0.947	-64.289



C = 2.4296 pFd
L = 9.06021 nH

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE1.OUT Tue Jan 31 14:31:13 1995

FREQ-GHZ	MAG[S11] DMN	ANG[S11] DMN	MAG[S21] DMN	ANG[S21] DMN	MAG[S22] DMN	ANG[S22] DMN
0.45000	0.999	2.239	0.048	-87.761	0.999	2.239
0.46000	0.999	2.178	0.047	-87.822	0.999	2.178
0.47000	0.999	2.119	0.046	-87.881	0.999	2.119
0.48000	0.999	2.062	0.046	-87.938	0.999	2.062
0.49000	0.999	2.007	0.045	-87.993	0.999	2.007
0.50000	0.999	1.954	0.044	-88.046	0.999	1.954
0.51000	0.999	1.903	0.044	-88.097	0.999	1.903
0.52000	0.999	1.853	0.043	-88.147	0.999	1.853
0.53000	0.999	1.805	0.042	-88.195	0.999	1.805
0.54000	0.999	1.758	0.042	-88.242	0.999	1.758
0.55000	0.999	1.713	0.041	-88.287	0.999	1.713

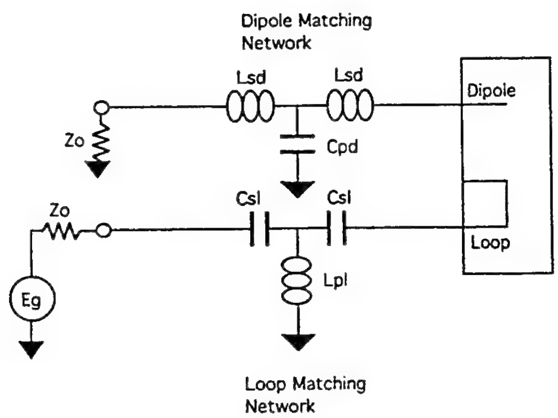


L = 406.0848 nH
C = .05707 pFd

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE1.OUT Tue Jan 31 16:36:06 1995

FREQ-GHZ	DB[S11] SPASSV	ANG[S11] SPASSV	DB[S22] SPASSV	ANG[S22] SPASSV	DB[S12] SPASSV	ANG[S12] SPASSV	DB[S21] SPASSV	ANG[S21] SPASSV
0.45000	-0.001	-13.547	-0.002	-52.916	-35.567	56.751	-35.179	56.750
0.46000	-0.004	-17.155	-0.004	-57.263	-30.847	52.769	-30.459	52.768
0.47000	-0.014	-23.273	-0.016	-63.904	-25.021	46.376	-24.633	46.376
0.48000	-0.088	-36.029	-0.091	-76.611	-17.217	33.622	-16.829	33.621
0.49000	-1.449	-77.663	-1.468	-115.861	-5.685	-6.895	-5.297	-6.896
0.50000	-47.450	-179.334	-73.786	-13.049	-0.212	-116.647	0.176	-116.647
0.51000	-1.811	77.831	-1.837	23.508	-4.887	140.818	-4.499	140.817
0.52000	-0.169	37.421	-0.176	-14.654	-14.412	101.468	-14.024	101.467
0.53000	-0.041	24.526	-0.045	-27.601	-20.557	88.523	-20.168	88.523
0.54000	-0.015	18.325	-0.018	-34.436	-24.789	81.993	-24.401	81.993
0.55000	-0.007	14.669	-0.010	-38.967	-27.937	77.894	-27.549	77.894

Circuit for Passive S-Parameter Calculation



Lsd = 406.0848 nH

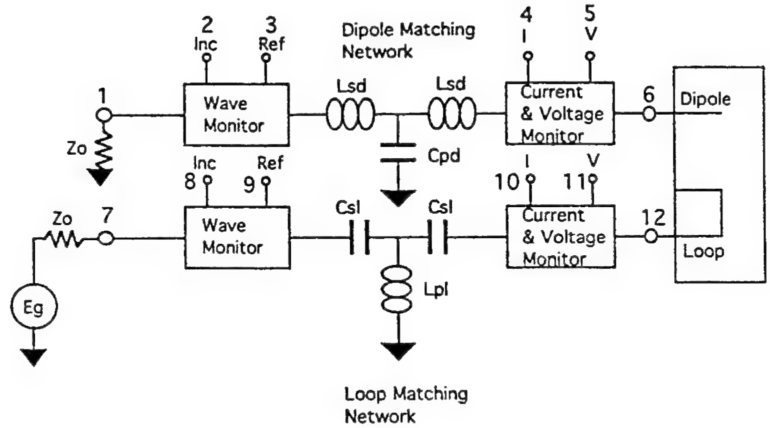
Cpd = .05707 pFd

Csl = 2.4296 pFd

Lpl = 9.06021 nH

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE1.OUT   Tue Jan 31 18:13:56 1995

FREQ-GHZ	RE[S21] ZANT Dipole	IM[S21] ZANT	RE[S31] ZANT Loop	IM[S31] ZANT
0.45000	-75.324	-2.6e+03	0.040	86.273
0.46000	-76.879	-2.6e+03	0.062	88.382
0.47000	-78.534	-2.7e+03	0.110	90.563
0.48000	-80.264	-2.8e+03	0.243	92.867
0.49000	-82.103	-2.9e+03	0.809	95.450
0.50000	-84.035	-2.9e+03	3.113	96.168
0.51000	-86.083	-3.0e+03	0.911	96.810
0.52000	-88.274	-3.1e+03	0.319	99.359
0.53000	-90.548	-3.2e+03	0.169	101.690
0.54000	-92.978	-3.3e+03	0.112	103.919
0.55000	-95.565	-3.3e+03	0.084	106.105



<u>From MathCad</u>

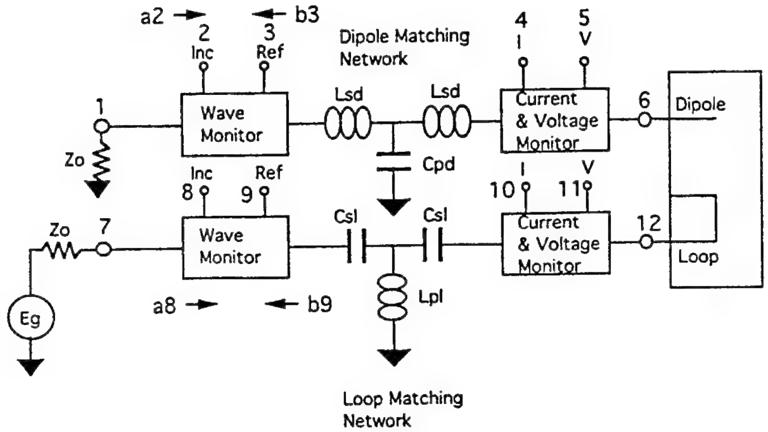
Zactive dipole = -84.045 - j 2928.9

Good agreement!

Zactive loop = 3.11211 + j 96.16673

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
                 MIXMODE1.OUT    Tue Jan 31 17:17:29 1995
```

FREQ-GHZ	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]
	INC	INC	INC	INC	REFL	REFL	REFL	REFL
	$a_2 \rightarrow$		$a_8 \rightarrow$		$\leftarrow b_3$		$\leftarrow b_9$	
0.45000	1.2e-09	-123.249	1.000	0.000	0.017	56.751	1.000	-52.916
0.46000	2.0e-09	-127.231	1.000	0.000	0.029	52.769	0.999	-57.263
0.47000	4.0e-09	-133.624	1.000	0.000	0.056	46.376	0.998	-63.904
0.48000	9.8e-09	-146.378	1.000	0.000	0.138	33.622	0.990	-76.611
0.49000	3.7e-08	173.105	1.000	0.000	0.520	-6.895	0.845	-115.861
0.50000	6.9e-08	63.353	1.000	0.000	0.976	-116.647	2.0e-04	-12.957
0.51000	4.1e-08	-39.182	1.000	0.000	0.570	140.818	0.809	23.508
0.52000	1.4e-08	-78.532	1.000	0.000	0.190	101.468	0.980	-14.654
0.53000	6.7e-09	-91.477	1.000	0.000	0.094	88.523	0.995	-27.601
0.54000	4.1e-09	-98.007	1.000	0.000	0.058	81.993	0.998	-34.436
0.55000	2.9e-09	-102.106	1.000	0.000	0.040	77.894	0.999	-38.967

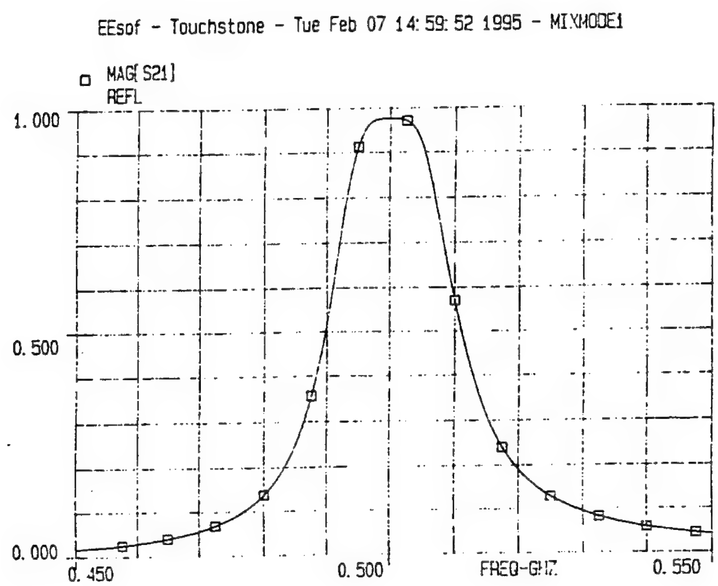


$|a_8|^2 = 1$ watt = input power

$|b_3|^2 = |0.976|^2 = 0.952576$ power dissipated in load

Power radiated = 0.047424 watts

$$\text{Efficiency} = \frac{\text{Power radiated}}{\text{Power input}} = 4.7\%$$



Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE1.OUT Tue Jan 31 17:32:02 1995

FREQ-GHZ	MAG[S21] VANT	ANG[S21] VANT	MAG[S31] VANT	ANG[S31] VANT	MAG[S21] IANT Dipole	ANG[S21] IANT	MAG[S31] IANT Loop	ANG[S31] IANT
0.45000	4.910	145.631	8.269	-26.563	0.002	-122.682	0.096	-116.537
0.46000	8.602	141.679	11.265	-28.827	0.003	-126.645	0.127	-118.786
0.47000	17.108	135.316	16.428	-32.391	0.006	-133.018	0.181	-122.321
0.48000	42.699	122.590	27.183	-39.691	0.015	-145.754	0.293	-129.541
0.49000	163.620	82.100	56.814	-65.914	0.057	173.750	0.595	-155.428
0.50000	311.924	-27.624	54.533	-117.854	0.106	64.019	0.567	154.000
0.51000	184.763	-130.134	59.568	-161.230	0.061	-38.495	0.615	109.309
0.52000	62.580	-169.459	35.070	174.079	0.020	-77.822	0.353	84.263
0.53000	31.268	177.620	24.981	166.679	0.010	-90.744	0.246	76.774
0.54000	19.461	171.114	20.011	163.008	0.006	-97.250	0.193	73.070
0.55000	13.715	167.038	17.105	160.646	0.004	-101.325	0.161	70.692

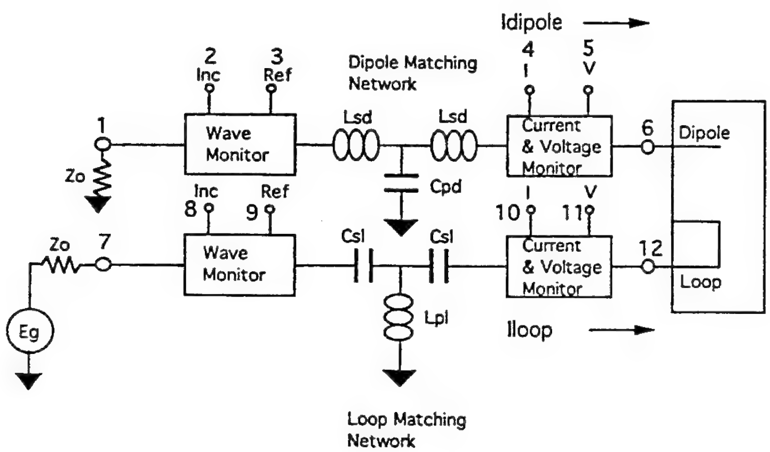
$$\frac{\text{Iloop}}{\text{Idipole}} = \frac{0.567 \quad 154}{0.106 \quad 64.019} = 5.349 \quad 89.91$$

From MathCad

$$\text{Iloop} = 5.09296 \quad 0^\circ$$

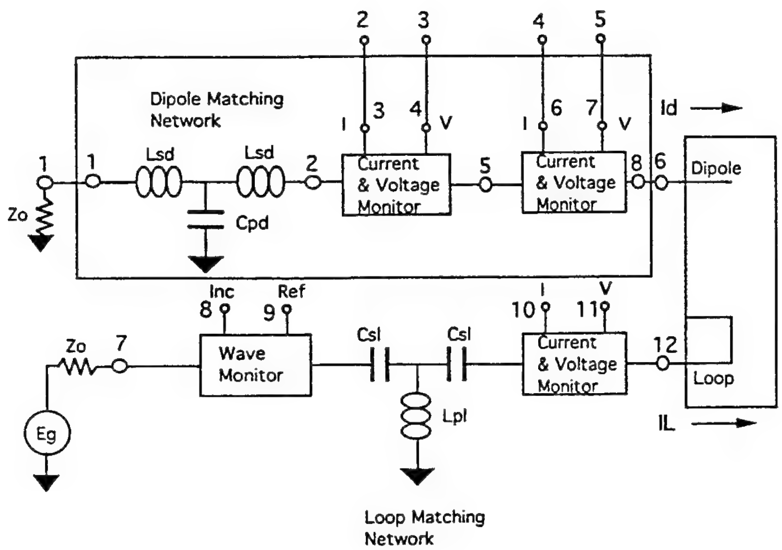
$$\text{Idipole} = 1.0000 \quad -90$$

$$\frac{\text{Iloop}}{\text{Idipole}} = 5.09296 \quad 90^\circ$$



```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
                 MIXMODE1.OUT    Tue Jan 31 18:37:28 1995
```

FREQ-GHZ	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]
	INORM	INORM	INORM	INORM
	Id/Id		IL/Id	
0.45000	1.000	7.5e-05	49.929	6.145
0.46000	1.000	1.1e-05	38.962	7.859
0.47000	1.000	-4.6e-04	28.655	10.697
0.48000	1.000	-6.1e-05	19.036	16.213
0.49000	1.000	5.7e-04	10.377	30.823
0.50000	1.000	4.5e-05	5.324	89.981
0.51000	1.000	4.6e-05	10.021	147.804
0.52000	1.000	-1.2e-04	17.427	162.085
0.53000	1.000	-8.1e-06	24.924	167.518
0.54000	1.000	-3.2e-04	32.229	170.320
0.55000	1.000	-4.9e-06	39.303	172.016

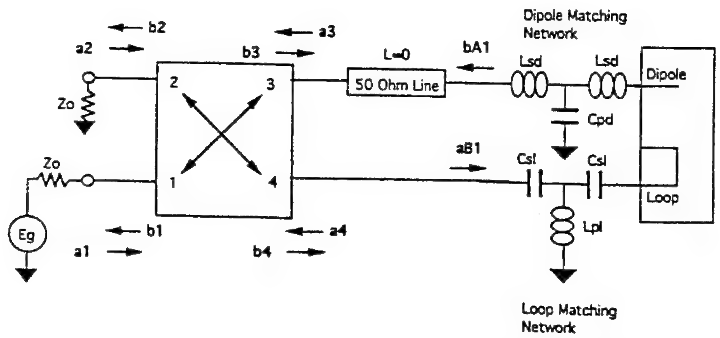


$$\frac{\text{Iloop}}{\text{Idipole}} = 5.324 \quad 89.981^\circ$$

Appendix G

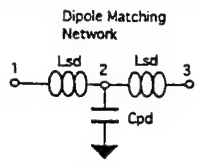
TOUCHSTONE ANALYSIS OF MIXED-MODE ARRAY (WEAK COUPLED FEEDBACK)

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
            MIXMODE2.CKT    Wed Feb 01 16:57:55 1995

DIM
  FREQ GHZ
  RES OH
  IND NH
  CAP PF
  LNG MIL
  TIME PS
  COND /OH
  ANG DEG
```

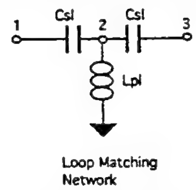



```
 ****************************************************************************
VAR
!**************************************************************************
!  VARIABLES ASSOCIATED WITH MIXED MODE ANTENNA
!**************************************************************************
  k = .21777     !Desired voltage coupling coefficient
  Zin = 50.      !Desired input impedance
!****************************************************************************
!DIPOLE MATCHING NETWORK (TEE SECTION)
!****************************************************************************
  LSD = 406.0848
  CPD = .05707
!****************************************************************************
!LOOP MATCHING NETWORK (TEE SECTION)
!****************************************************************************
  CSL = 2.4296
  LPL = 9.06021
!****************************************************************************
EQN
!****************************************************************************
  Zoe = SQRT((1-k)/(1+k))*Zin
  Zoo = SQRT((1+k)/(1-k))*Zin
CKT
!****************************************************************************
! Antenna Y-matrix (passive) from NEC MOM
  S2PA 1 2 0 mixmode2.S2P
  DEF2P 1 2 ANTENNA
! current & voltage monitor
  S4PA 1 2 3 4 IVMETER.S4P
  DEF4P 1 2 3 4 IVMETER
```

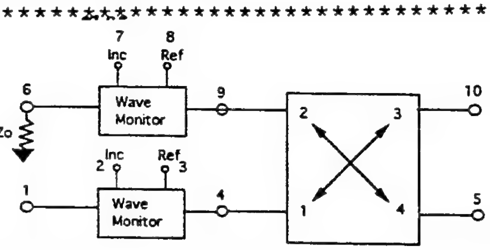
```
! reflection coefficient monitor
  S4PB 1 2 3 4 REFLMETR.S4P
  DEF4P 1 2 3 4 REFLMON
!*******************************************************************
! Dipole Matching Network
!*******************************************************************
  IND 1 2 L^LSD
  CAP 2 0 C^CPD
  IND 2 3 L^LSD
  DEF2P 1 3 DMN
```



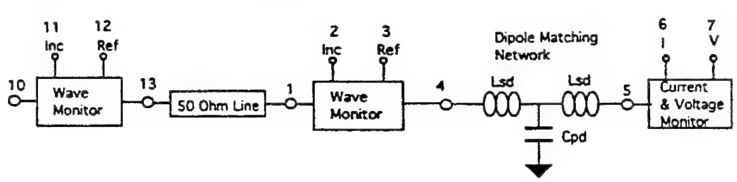

```
!*******************************************************************
! Loop Matching Network
!*******************************************************************
  CAP 1 2 C^CSL
  IND 2 0 L^LPL
  CAP 2 3 C^CSL

  DEF2P 1 3 LMN
```



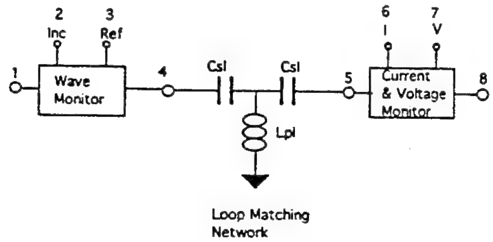

```
!*******************************************************************
  Ideal Directional Coupler
!*******************************************************************
  CLIN 1 2 3 4 ZE^Zoe Zo^Zoo E=90 F=.5
  DEF4P   1 2 3 4 COUPLER
*******************************************************************
! Define 7-port Coupler and wave monitor network
*******************************************************************
  REFLMON 1 4 2 3
  COUPLER 4 9 10 5
  REFLMON 6 9 7 8
  RES 6 0 R=50
  DEF7P 1 2 3 5 7 8 10   CPLMON
```




```
!*******************************************************************
! Define 8-port Channel #1  (DIPOLE)
!*******************************************************************
  REFLMON 10 13 11 12
  TLIN 13 1 Z=50 E=0 F=.5
  REFLMON 1 4 2 3
  DMN 4 5
  IVMETER 5 8 6 7
  DEF8P 10 11 12  2 3 6 7 8 CHAN1
```

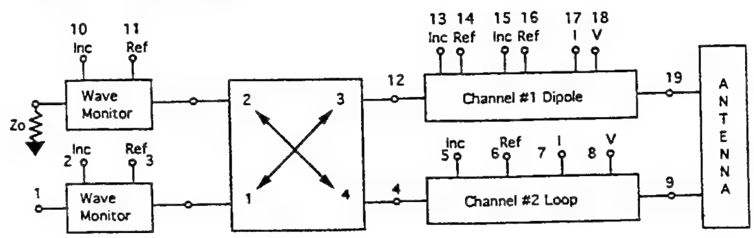


```
!*************************************************************************
! Define 6-port Channel #2  (LOOP)
!*************************************************************************
 REFLMON 1 4 2 3
 LMN 4 5
 IVMETER 5 8 6 7
 DEF6P 1 2 3 6 7 8 CHAN2
```




```
!*************************************************************************
! Define 2-port to measure Passive S-Parameters of antenna and matching NW
!*************************************************************************
 DMN 1 2
 LMN 4 3
 ANTENNA 2 3
 DEF2P 1 4  SPASSV
!*************************************************************************
! Define 3-port to measure Reflected waves at input to matching Networks
! Select REFL S21 to measure reflected wave in channel 1 (DIPOLE)
! Select REFL S31 to measure reflected wave in channel 2 (LOOP)
!*************************************************************************

 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 16 6 REFL
!*************************************************************************
! Define 3-port to measure Incident waves at input to matching Networks
! Select INC S21 to measure incident wave in channel 1 (DIPOLE)
! Select INC S31 to measure incident wave in channel 2 (LOOP)
!*************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5 -6  7 8 9

 ANTENNA 19 9
 DEF3P 1 15 5 INC
*************************************************************************
! Define 3-port to measure VANT (voltage at antenna) for all Channels
' Select VANT S21 for Voltage at DIPOLE
  Select VANT S31 for Voltage at LOOP
.*************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 18 8 VANT
*************************************************************************
! Define 3-port to measure IANT (current AT antenna) for all Channels
' Select IANT S21 for current at DIPOLE
  Select IANT S31 for current at LOOP
:*************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 17 7 IANT
```

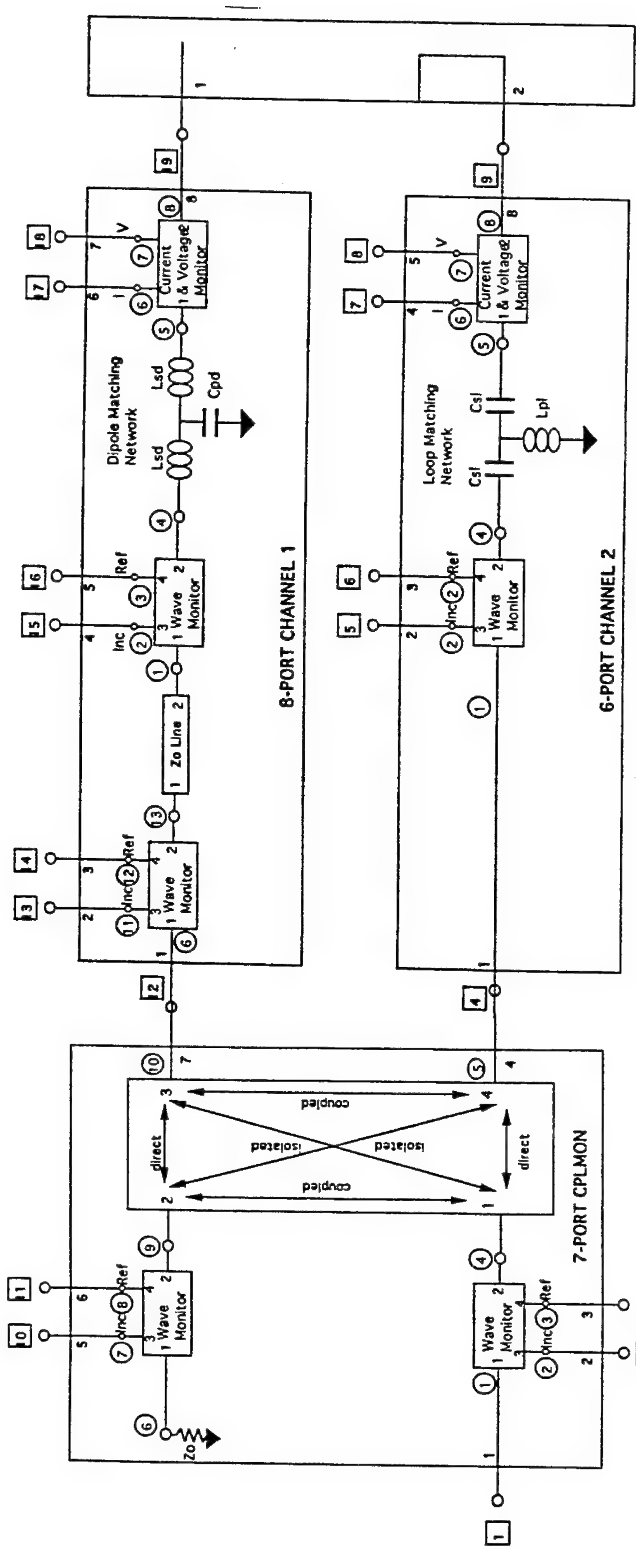
```
***********************************************************************
! Define 6-port Current Reference Channel (#1) DIPOLE
***********************************************************************
 TLIN 13 1 Z=50 E=0 F=.5
 DMN 1 5
 IVMETER 5 8 6 7
 IVMETER 8 9 10 11
 DEF6P 13 6 7  10 11  9 CHANREF
!***********************************************************************
! Define 3 port current reference (Normalize antenna currents to DIPOLE)
!***********************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHANREF  12 13 14 15  16  19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 13 15 IREF
***********************************************************************
! Define 5 port to measure incident waves at coupler ports
! Select S21 for incident wave at port 1 of coupler
! Select S31 for incident wave at port 2 of coupler
! Select S41 for incident wave at port 3 of coupler
! Select S51 for incident wave at port 4 of coupler
!***********************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF5P 1 2 10  14 6   CPLINC
```




```
!***********************************************************************

! Define 5 port to measure reflected waves at coupler ports
! Select S21 for reflected wave at port 1 of coupler
! Select S31 for reflected wave at port 2 of coupler
! Select S41 for reflected wave at port 3 of coupler
! Select S51 for reflected wave at port 4 of coupler
!***********************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF5P 1 3 11  13 5   CPLREF
TERM
?ROC
 GAMMA = REFL / INC
 ZANT  = VANT / IANT
 INORM = IANT / IREF
```

```
OUT
!Passive Antenna Data
!  ANTENNA MAG[S11]
!  ANTENNA ANG[S11]
!  ANTENNA MAG[S12]
!  ANTENNA ANG[S12]
!  ANTENNA MAG[S21]
!  ANTENNA ANG[S21]
!  ANTENNA MAG[S22]
!  ANTENNA ANG[S22]
!Passive matching network S-paramters
! DMN MAG[S11]
! DMN ANG[S11]
! LMN MAG[S11]
! LMN ANG[S11]
! DMN MAG[S21]
! DMN ANG[S21]
! LMN MAG[S21]
! LMN ANG[S21]
! DMN MAG[S22]
! DMN ANG[S22]
! LMN MAG[S22]
! LMN ANG[S22]
! Passive Scattering Parameters of Antenna and Matching Networks
! SPASSV DB[S11] GR1
! SPASSV ANG[S11] GR2
! SPASSV DB[S22]  GR1
! SPASSV ANG[S22] GR2
! SPASSV DB[S12]  GR1
! SPASSV ANG[S12] GR2
! SPASSV DB[S21]  GR1
! SPASSV ANG[S21] GR2
!Reflection coefficients
!   GAMMA S21 SC2
!   GAMMA S31 SC2
!Incident waves
    INC MAG[S21]
    INC ANG[S21]
    INC MAG[S31]
    INC ANG[S31]
    REFL MAG[S21]
    REFL ANG[S21]
    REFL MAG[S31]
    REFL ANG[S31]
'Antenna Voltages
    VANT S21
!   VANT S31
!Antenna Currents
    IANT S21
.   IANT S31
!Active impedances
   ZANT  RE[S21]
   ZANT  IM[S21]
!  ZANT  RE[S31]
!  ZANT  IM[S31]
 Normalized Current Ratios
!  Inorm MAG[S21]
!  Inorm ANG[S21]
:  Inorm MAG[S31]
:  Inorm ANG[S31]
```

```
!Coupler S-parameters
:  COUPLER MAG[S11]
:  COUPLER ANG[S11]
!  COUPLER MAG[S22]
:  COUPLER ANG[S22]
!  COUPLER MAG[S33]
!  COUPLER ANG[S33]
!  COUPLER MAG[S44]
!  COUPLER ANG[S44]
!  COUPLER MAG[S12]
!  COUPLER ANG[S12]
!  COUPLER MAG[S21]
!  COUPLER ANG[S21]
!  COUPLER MAG[S34]
!  COUPLER ANG[S34]
!  COUPLER MAG[S43]
!  COUPLER ANG[S43]
!  COUPLER MAG[S13]
!  COUPLER ANG[S13]
!  COUPLER MAG[S31]
!  COUPLER ANG[S31]
!  COUPLER MAG[S24]
!  COUPLER ANG[S24]
!  COUPLER MAG[S42]
!  COUPLER ANG[S42]
!  COUPLER MAG[S14]
!  COUPLER ANG[S14]
!  COUPLER MAG[S41]
!  COUPLER ANG[S41]
!  COUPLER MAG[S23]
!  COUPLER ANG[S23]
!  COUPLER MAG[S32]
!  COUPLER ANG[S32]
!  CPLINC MAG[S21]
!  CPLINC ANG[S21]
!  CPLINC MAG[S31]
!  CPLINC ANG[S31]
!  CPLINC MAG[S41]
!  CPLINC ANG[S41]
   CPLINC MAG[S51]
   CPLINC ANG[S51]
!  CPLREF MAG[S21]
'  CPLREF ANG[S21]
   CPLREF MAG[S31]
!  CPLREF ANG[S31]
!  CPLREF MAG[S41]
   CPLREF ANG[S41]
.  CPLREF MAG[S51]
!  CPLREF ANG[S51]
'REQ
   SWEEP .450 .550 .010
!  STEP .500
GRID
  RANGE .450 .550 .010
  GR1 0 -60 10
  GR2 180 -180 30
)PT
```

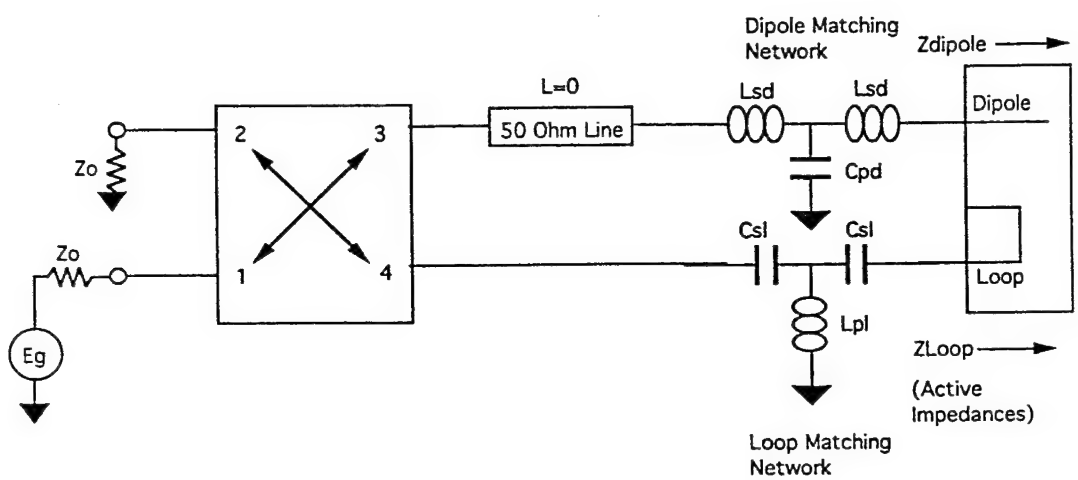


Key

N Denotes the node numbers of final Touchstone Circuit
N Denotes the numbering sequence of defined multiports
N Denotes the internal node numbers of multiports

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE2.OUT   Wed Feb 01 17:15:13 1995

FREQ-GHZ	RE[S21] ZANT Dipole	IM[S21] ZANT	RE[S31] ZANT Loop	IM[S31] ZANT
0.45000	-62.636	-3.1e+03	0.360	87.001
0.46000	-71.491	-3.0e+03	0.438	89.058
0.47000	-78.804	-3.0e+03	0.550	91.160
0.48000	-83.363	-2.9e+03	0.754	93.325
0.49000	-84.043	-2.9e+03	1.318	95.587
0.50000	-84.038	-2.9e+03	3.113	96.168
0.51000	-132.121	-3.1e+03	0.558	97.326
0.52000	-645.136	-3.4e+03	0.271	100.083
0.53000	-1.1e+03	-2.6e+03	0.282	102.412
0.54000	-839.964	-2.3e+03	0.316	104.611
0.55000	-717.800	-2.2e+03	0.350	106.763



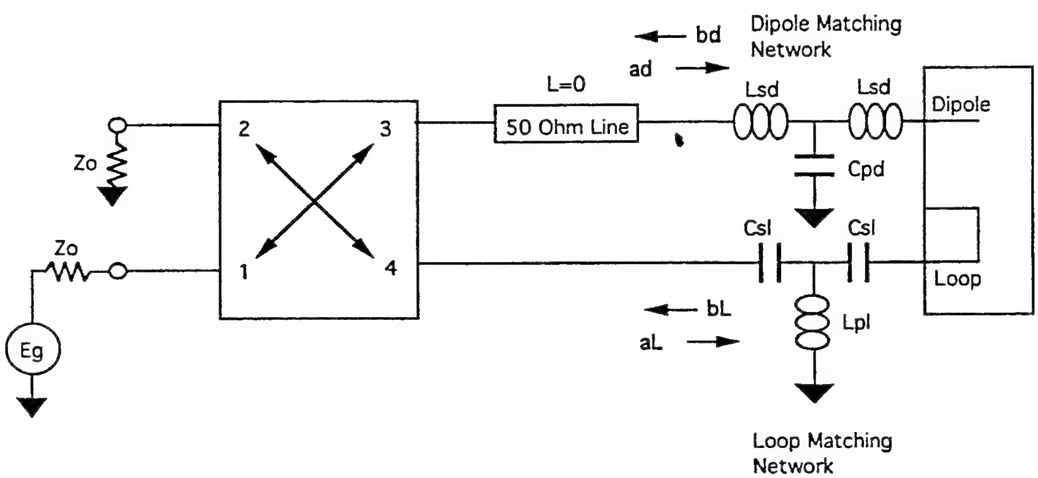
Compare with active impedance from 50 Ω case

Dipole

-84.035	-2,900	3.113	96.168

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
            MIXMODE2.OUT    Wed Feb 01 16:54:03 1995
```

FREQ-GHZ	MAG[S21] INC	ANG[S21] INC	MAG[S31] INC	ANG[S31] INC	MAG[S21] REFL	ANG[S21] REFL	MAG[S31] REFL	ANG[S31] REFL
	$\vec{a}_d$		$\vec{a}_L$		$\overleftarrow{b}_d$		$\overleftarrow{b}_L$	
0.45000	0.216	52.211	1.005	-83.465	0.223	34.757	1.003	-136.576
0.46000	0.214	43.793	0.995	-85.653	0.230	20.497	0.991	-143.236
0.47000	0.210	32.922	0.977	-87.878	0.248	-0.280	0.968	-152.349
0.48000	0.197	15.975	0.940	-89.871	0.309	-34.366	0.907	-167.539
0.49000	0.140	-21.046	0.854	-87.485	0.563	-95.295	0.645	157.197
0.50000	5.0e-05	102.086	1.056	-78.142	1.030	165.211	2.3e-04	-77.981
0.51000	0.209	101.654	1.074	-94.716	0.510	60.069	0.959	-76.589
0.52000	0.220	64.013	1.027	-95.352	0.277	56.863	1.012	-112.474
0.53000	0.219	48.731	1.019	-97.190	0.250	51.037	1.011	-125.998
0.54000	0.218	38.658	1.013	-99.159	0.240	43.451	1.007	-134.313
0.55000	0.216	30.666	1.007	-101.098	0.233	36.070	1.003	-140.547

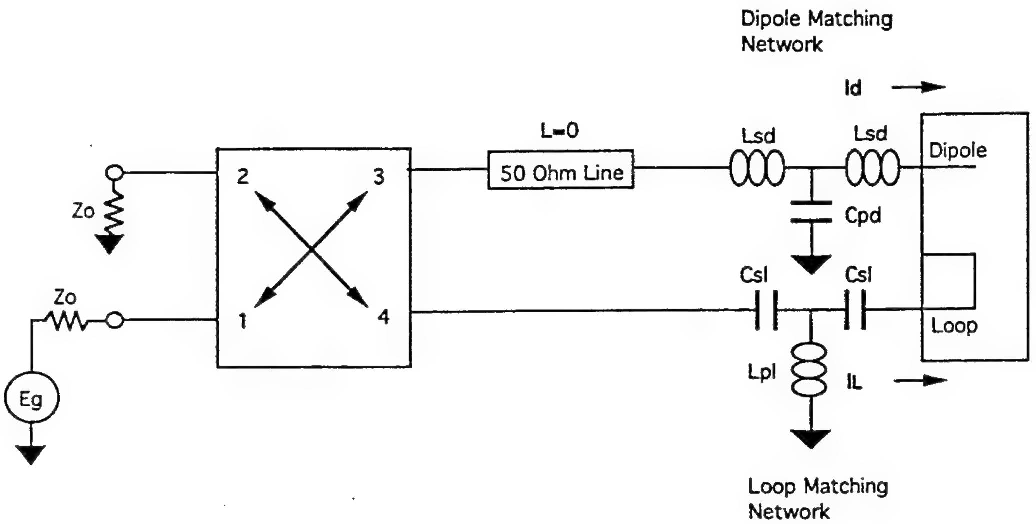


Note $\overleftarrow{b}_L$ and $\vec{a}_d$ are zero as expected

$|\vec{a}_L|$ is > 1 because of the additional coupled power

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
            MIXMODE2.OUT    Wed Feb 01 17:05:35 1995
```

FREQ-GHZ	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]
	VANT	VANT	VANT	VANT	IANT	IANT	IANT	IANT
	V_d		V_L		I_d		I_L	
0.45000	22.242	49.073	8.585	-110.984	0.007	140.240	0.099	159.253
0.46000	29.723	41.034	11.613	-115.847	0.010	132.395	0.130	154.435
0.47000	43.733	30.849	16.674	-122.370	0.015	122.378	0.183	147.976
0.48000	75.999	15.021	26.376	-133.156	0.026	106.662	0.283	137.308
0.49000	172.044	-20.758	46.606	-158.329	0.059	70.905	0.488	112.460
0.50000	329.322	-105.764	57.578	164.002	0.112	-14.120	0.598	75.856
0.51000	125.379	127.297	62.873	98.490	0.041	-140.256	0.646	8.819
0.52000	22.780	116.147	34.220	77.210	0.007	-142.996	0.342	-12.635
0.53000	10.353	143.669	24.547	69.167	0.004	-103.605	0.240	-20.675
0.54000	8.544	159.661	19.750	63.882	0.003	-90.341	0.189	-25.945
0.55000	7.798	162.745	16.899	59.700	0.003	-89.451	0.158	-30.112



Compare with results with no coupler (terminated in 50 Ω)

V_d		V_L		I_d		I_L	
311.924	-27.624	54.533	-117.854	0.106	64.019	0.567	154.000

Coupler

$$\frac{I_{loop}}{I_{dipole}} = \frac{0.598 \quad 75.856}{0.112 \quad -14.120}$$

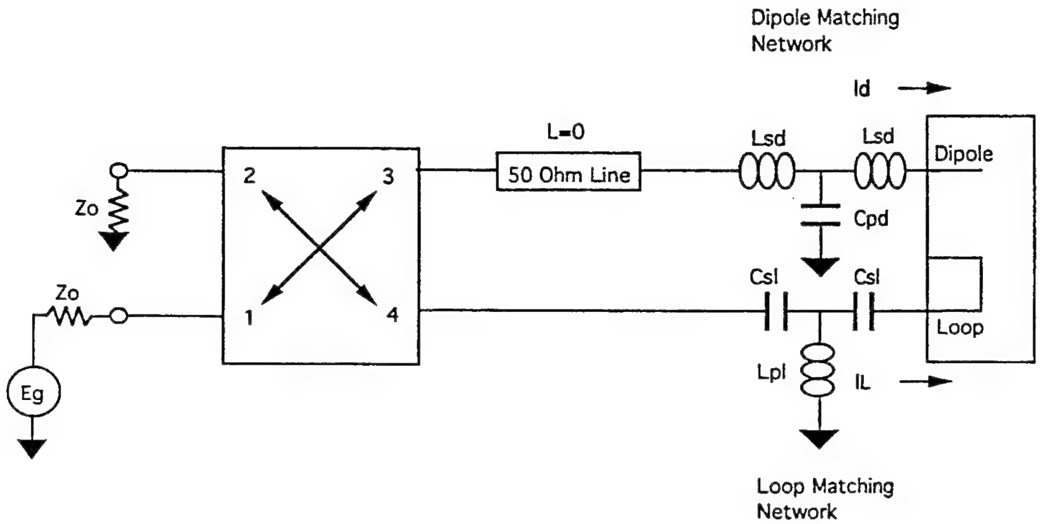
$$= 5.339 \quad 89.976$$

50 r load

$$\frac{0.567 \quad 154}{0.106 \quad 64.019} = 5.349 \quad 89.98$$

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE2.OUT Wed Feb 01 17:19:54 1995

FREQ-GHZ	MAG[S21] INORM	ANG[S21] INORM	MAG[S31] INORM	ANG[S31] INORM
	Id/Id		IL/Id	
0.45000	1.000	-6.2e-05	13.643	19.013
0.46000	1.000	-2.0e-04	13.203	22.039
0.47000	1.000	-7.3e-04	12.353	25.597
0.48000	1.000	-4.9e-04	10.830	30.645
0.49000	1.000	-9.1e-05	8.205	41.555
0.50000	1.000	2.5e-04	5.324	89.976
0.51000	1.000	2.9e-04	15.944	149.075
0.52000	1.000	2.3e-04	51.407	130.361
0.53000	1.000	7.4e-05	65.902	82.930
0.54000	1.000	-4.2e-04	54.269	64.395
0.55000	1.000	-4.0e-05	47.650	59.340

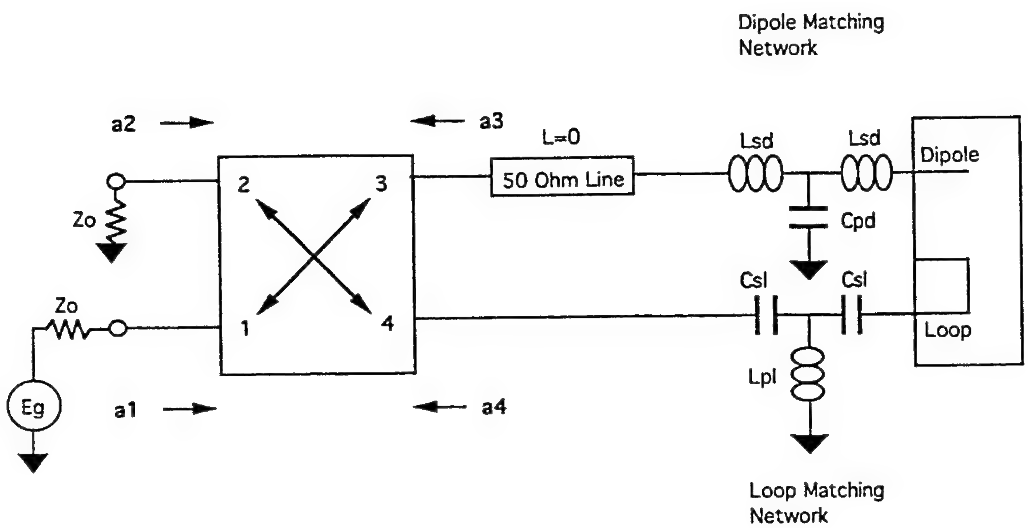


Compare with 50 r results

Id/Id		IL/Id	
1.000	4.5e-05	5.324	89.981

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
                  MIXMODE2.OUT    Wed Feb 01 17:39:35 1995
```

FREQ-GHZ	MAG[S21] CPLINC	ANG[S21] CPLINC	MAG[S31] CPLINC	ANG[S31] CPLINC	MAG[S41] CPLINC	ANG[S41] CPLINC	MAG[S31] REFL	ANG[S31] REFL
		a_1 →		a_2 →	← a_3		← a_4	
0.45000	1.000	0.000	1.4e-08	71.892	0.223	34.757	1.003	-136.576
0.46000	1.000	0.000	1.8e-08	63.841	0.230	20.497	0.991	-143.236
0.47000	1.000	0.000	2.3e-08	53.225	0.248	-0.280	0.968	-152.349
0.48000	1.000	0.000	3.3e-08	36.240	0.309	-34.366	0.907	-167.539
0.49000	1.000	0.000	5.5e-08	-2.036	0.563	-95.295	0.645	157.197
0.50000	1.000	0.000	6.9e-08	-92.293	1.030	165.211	2.3e-04	-77.981
0.51000	1.000	0.000	2.3e-08	128.976	0.510	60.069	0.959	-76.589
0.52000	1.000	0.000	1.1e-08	90.071	0.277	56.863	1.012	-112.474
0.53000	1.000	0.000	1.1e-08	74.789	0.250	51.037	1.011	-125.998
0.54000	1.000	0.000	1.3e-08	65.031	0.240	43.451	1.007	-134.313
0.55000	1.000	0.000	1.4e-08	57.478	0.233	36.070	1.003	-140.547

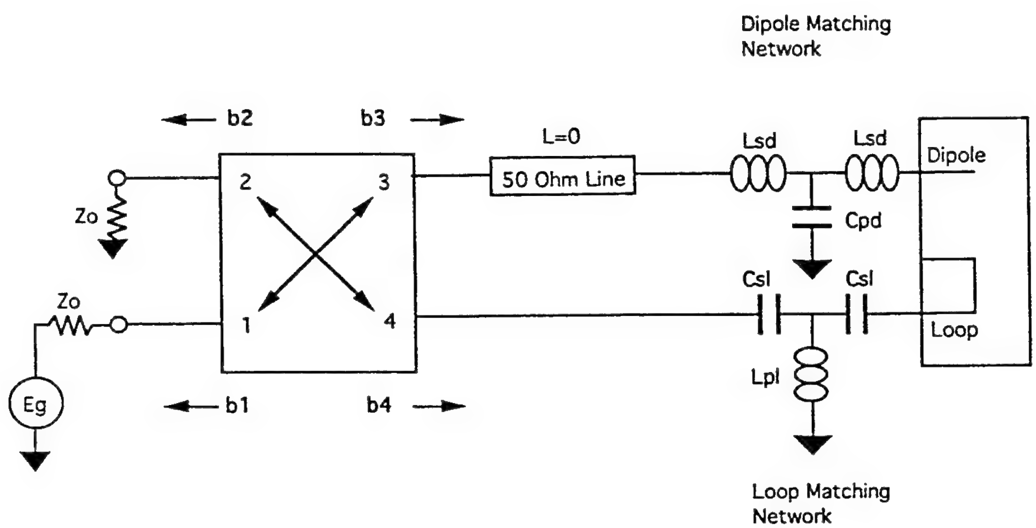


a_2 and a_4 are zero as expected

$|a_3| > 1.0$ interesting!
$|a_1| = 1.0$

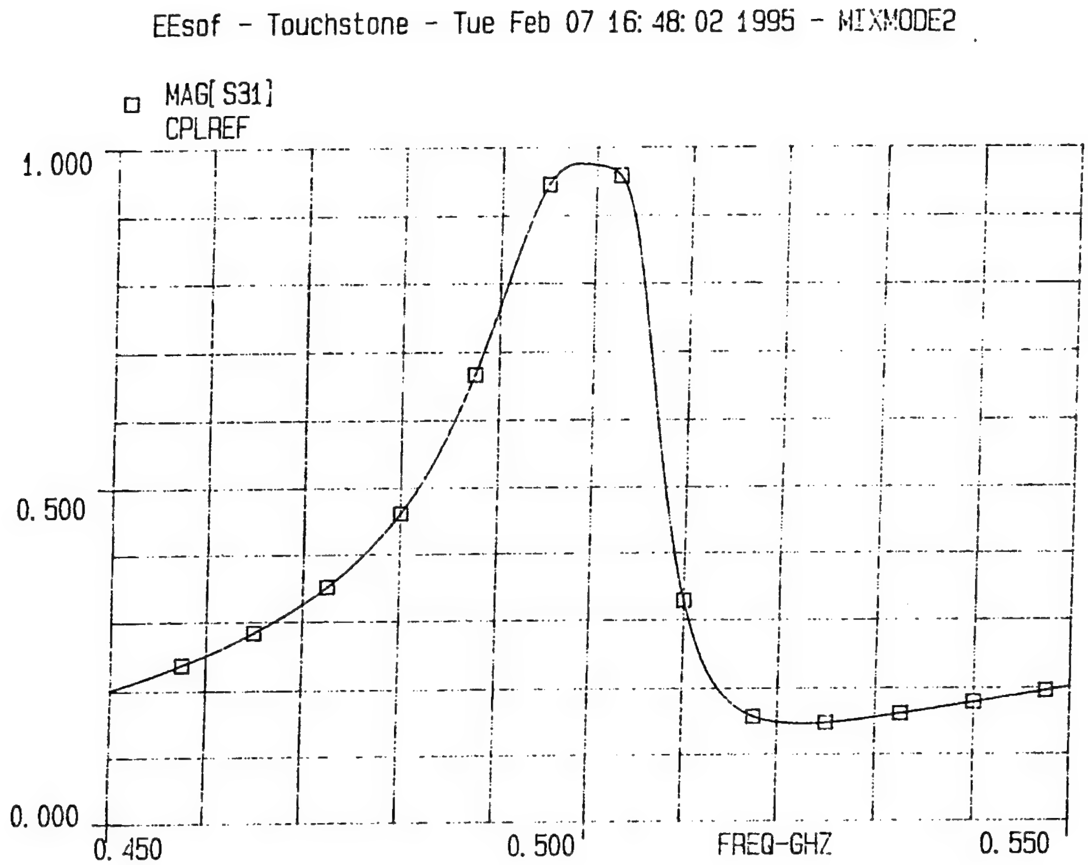
Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE2.OUT   Wed Feb 01 17:52:29 1995

FREQ-GHZ	MAG[S21] CPLREF	ANG[S21] CPLREF	MAG[S31] CPLREF	ANG[S31] CPLREF	MAG[S41] CPLREF	ANG[S41] CPLREF	AG[S31] INC	ANG[S31] INC
	b_1		b_2		b_3		a_L	
0.45000	0.979	142.211	0.201	-108.108	0.216	52.211	1.005	-83.465
0.46000	0.968	133.793	0.251	-116.159	0.214	43.793	0.995	-85.653
0.47000	0.945	122.922	0.326	-126.775	0.210	32.922	0.977	-87.878
0.48000	0.885	105.975	0.461	-143.760	0.197	15.975	0.940	-89.871
0.49000	0.630	68.954	0.766	177.964	0.140	-21.046	0.854	-87.485
0.50000	2.2e-04	-167.982	0.973	87.707	5.0e-05	102.086	1.056	-78.142
0.51000	0.936	-168.346	0.328	-51.024	0.209	101.654	1.074	-94.716
0.52000	0.987	154.013	0.148	-89.929	0.220	64.013	1.027	-95.352
0.53000	0.987	138.731	0.156	-105.211	0.219	48.731	1.019	-97.190
0.54000	0.983	128.658	0.179	-114.969	0.218	38.658	1.013	-99.159
0.55000	0.979	120.666	0.201	-122.522	0.216	30.666	1.007	-101.098



b_2 is **not** zero !

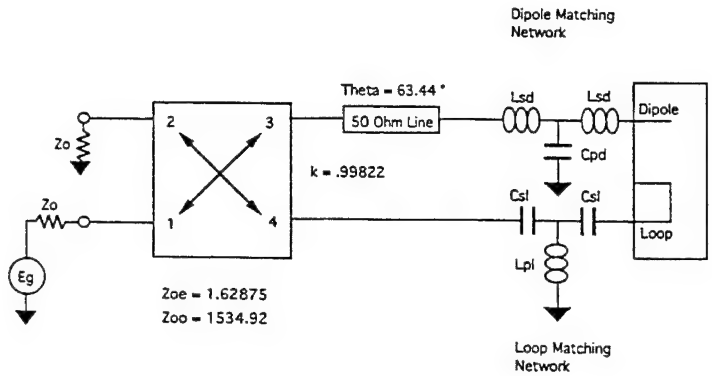
Need to try to adjust 50 ohm line length to force $b_2 \to 0$


EEsof - Touchstone - Tue Feb 07 16:48:02 1995 - MIXMODE2
MAG[S31]
CPLREF
1.000
0.500
0.000
0.450
0.500
FREQ-GHZ
0.550

Appendix H

TOUCHSTONE ANALYSIS OF MIXED-MODE ARRAY (OPTIMUM FEEDBACK - DETERMINED BY TOUCHSTONE OPT.)

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
             MIXMODE3.CKT    Mon Feb 06 12:52:42 1995

DIM
  FREQ GHZ
  RES OH
  IND NH
  CAP PF
  LNG MIL
  TIME PS
  COND /OH
  ANG DEG
```




```
**********************************************************************
VAR
'*****************************************************************
  VARIABLES ASSOCIATED WITH MIXED MODE ANTENNA
!*****************************************************************
  k # .996  0.99822 1.0          ! .99788   !Desired voltage coupling coeffici
  Zin = 50.     !Desired input impedance
  LL1 #  60. 63.44028  65.   !89.44003  !Desired differential phase shift (i
!**********************************************************************
 DIPOLE MATCHING NETWORK (TEE SECTION)
.**********************************************************************
  LSD = 408.61049
  CPD = .05459
 *********************************************************************
!LOOP MATCHING NETWORK (TEE SECTION)
'**********************************************************************
  CSL = 2.42014
  LPL = 9.17388
!**********************************************************************
:QN
.**********************************************************************
  Zoe = SQRT((1-k)/(1+k))*Zin                1.62875
  Zoo = SQRT((1+k)/(1-k))*Zin
:KT                                          1534.92
:**********************************************************************
```

```
Antenna Y-matrix (passive) from NEC MOM
S2PA 1 2 0 mixmode3.S2P
DEF2P 1 2 ANTENNA
current & voltage monitor
S4PA 1 2 3 4 IVMETER.S4P
DEF4P 1 2 3 4 IVMETER
reflection coefficient monitor
S4PB 1 2 3 4 REFLMETR.S4P
DEF4P 1 2 3 4 REFLMON
**********************************************************************
 Dipole Matching Network
**********************************************************************
 IND 1 2 L^LSD
 CAP 2 0 C^CPD
 IND 2 3 L^LSD
 DEF2P 1 3 DMN
**********************************************************************
 Loop Matching Network
**********************************************************************
 CAP 1 2 C^CSL
 IND 2 0 L^LPL
 CAP 2 3 C^CSL
 DEF2P 1 3 LMN
!*********************************************************************
! Ideal Directional Coupler
!*********************************************************************
  CLIN 1 2 3 4 ZE^Zoe Zo^Zoo E=90. F=.5
  DEF4P  1 2 3 4 COUPLER
!*********************************************************************
! Define 7-port Coupler and wave monitor network
!*********************************************************************
  REFLMON 1 4 2 3
  COUPLER 4 9 10 5
  REFLMON 6 9 7 8
  RES 6 0 R=50
  DEF7P 1 2 3 5 7 8 10  CPLMON
!*********************************************************************
! Define 8-port Channel #1  (DIPOLE)
!*********************************************************************
  REFLMON 10 13 11 12
  TLIN 13 1 Z=50 E^LL1 F=.5
  REFLMON 1 4 2 3
  DMN 4 5
  IVMETER 5 8 6 7
  DEF8P 10 11 12  2 3 6 7 8 CHAN1
!*********************************************************************
! Define 6-port Channel #2  (LOOP)
!*********************************************************************
  REFLMON 1 4 2 3
  LMN 4 5
  IVMETER 5 8 6 7
  DEF6P 1 2 3 6 7 8 CHAN2
!*********************************************************************
! Define 2-port to measure Passive S-Parameters of antenna and matching NW
!*********************************************************************
  DMN 1 2
  LMN 4 3
  ANTENNA 2 3
  DEF2P 1 4  SPASSV
!*********************************************************************
```

```
! Define 3-port to measure Reflected waves at input to matching Networks
! Select REFL S21 to measure reflected wave in channel 1 (DIPOLE)
! Select REFL S31 to measure reflected wave in channel 2 (LOOP)
!**************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 16 6 REFL
**************************************************************************
 Define 3-port to measure Incident waves at input to matching Networks
 Select INC S21 to measure incident wave in channel 1 (DIPOLE)
 Select INC S31 to measure incident wave in channel 2 (LOOP)
**************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 15 5 INC
!**************************************************************************
! Define 3-port to measure VANT (voltage at antenna) for all Channels
  Select VANT S21 for Voltage at DIPOLE
. Select VANT S31 for Voltage at LOOP
!**************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 18 8 VANT
!**************************************************************************
! Define 3-port to measure IANT (current AT antenna) for all Channels
  Select IANT S21 for current at DIPOLE
: Select IANT S31 for current at LOOP
!**************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 17 7 IANT
!**************************************************************************
! Define 6-port Current Reference Channel (#1) DIPOLE
**************************************************************************
 TLIN 13 1 Z=50 E^LL1 F=.5
 DMN 1 5
 IVMETER 5 8 6 7
 IVMETER 8 9 10 11
 DEF6P 13 6 7  10 11  9 CHANREF
!**************************************************************************
. Define 3 port current reference (Normalize antenna currents to DIPOLE)
!**************************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHANREF  12 13 14 15  16  19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF3P 1 13 15 IREF
**************************************************************************
```

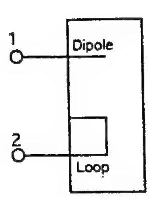
```
 Define 5 port to measure incident waves at coupler ports
 Select S21 for incident wave at port 1 of coupler
 Select S31 for incident wave at port 2 of coupler
 Select S41 for incident wave at port 3 of coupler
. Select S51 for incident wave at port 4 of coupler
***********************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF5P 1 2 10  14 6   CPLINC
***********************************************************************
 Define 5 port to measure reflected waves at coupler ports
 Select S21 for reflected wave at port 1 of coupler
 Select S31 for reflected wave at port 2 of coupler
! Select S41 for reflected wave at port 3 of coupler
: Select S51 for reflected wave at port 4 of coupler
***********************************************************************
 CPLMON 1  2  3  4  10  11 12
 CHAN1 12 13 14 15  16  17 18 19
 CHAN2 4  5  6  7 8 9
 ANTENNA 19 9
 DEF5P 1 3 11  13 5   CPLREF
:ERM
?ROC
 GAMMA = REFL / INC
 ZANT   = VANT / IANT
 INORM = IANT / IREF
OUT
!Passive Antenna Data
:  ANTENNA MAG[S11]
!  ANTENNA ANG[S11]
!  ANTENNA MAG[S12]
:  ANTENNA ANG[S12]
:  ANTENNA MAG[S21]
!  ANTENNA ANG[S21]
:  ANTENNA MAG[S22]
:  ANTENNA ANG[S22]
!Passive matching network S-paramters
: DMN MAG[S11]
: DMN ANG[S11]
! LMN MAG[S11]
! LMN ANG[S11]
! DMN MAG[S21]
: DMN ANG[S21]
  LMN MAG[S21]
  LMN ANG[S21]
  DMN MAG[S22]
  DMN ANG[S22]
  LMN MAG[S22]
  LMN ANG[S22]
  Passive Scattering Parameters of Antenna and Matching Networks
  SPASSV DB[S11] GR1
  SPASSV ANG[S11] GR2
  SPASSV DB[S22]  GR1
  SPASSV ANG[S22] GR2
  SPASSV DB[S12]  GR1
  SPASSV ANG[S12] GR2
  SPASSV DB[S21]  GR1
  SPASSV ANG[S21] GR2
```

```
Reflection coefficients
   GAMMA S21 SC2
   GAMMA S31 SC2
Incident waves
   INC MAG[S21]
   INC ANG[S21]
   INC MAG[S31]
   INC ANG[S31]
   REFL MAG[S21]
   REFL ANG[S21]
   REFL MAG[S31]
   REFL ANG[S31]
Antenna Voltages
   VANT S21
   VANT S31
Antenna Currents
   IANT S21
   IANT S31
Active impedances
   ZANT  RE[S21]
   ZANT  IM[S21]
   ZANT  RE[S31]
   ZANT  IM[S31]
Normalized Current Ratios
   Inorm MAG[S21]
   Inorm ANG[S21]
   Inorm MAG[S31]
   Inorm ANG[S31]
Coupler S-parameters
   COUPLER MAG[S11]
   COUPLER ANG[S11]
   COUPLER MAG[S22]
   COUPLER ANG[S22]
   COUPLER MAG[S33]
   COUPLER ANG[S33]
   COUPLER MAG[S44]
   COUPLER ANG[S44]
   COUPLER MAG[S12]
   COUPLER ANG[S12]
   COUPLER MAG[S21]
   COUPLER ANG[S21]
   COUPLER MAG[S34]
   COUPLER ANG[S34]
   COUPLER MAG[S43]
   COUPLER ANG[S43]
   COUPLER MAG[S13]
   COUPLER ANG[S13]
   COUPLER MAG[S31]
   COUPLER ANG[S31]
   COUPLER MAG[S24]
   COUPLER ANG[S24]
   COUPLER MAG[S42]
   COUPLER ANG[S42]
   COUPLER MAG[S14]
   COUPLER ANG[S14]
   COUPLER MAG[S41]
   COUPLER ANG[S41]
   COUPLER MAG[S23]
   COUPLER ANG[S23]
   COUPLER MAG[S32]
   COUPLER ANG[S32]
```

```
  CPLINC MAG[S21]
  CPLINC ANG[S21]
  CPLINC MAG[S31]
  CPLINC ANG[S31]
  CPLINC MAG[S41]
  CPLINC ANG[S41]
  CPLINC MAG[S51]   5-PORT S-PARAMETERS NOT ALLOWED
  CPLINC ANG[S51]   5-PORT S-PARAMETERS NOT ALLOWED
   CPLREF DB[S21]  GR1
   CPLREF ANG[S21]
   CPLREF DB[S31]  GR1
   CPLREF ANG[S31]
   CPLREF MAG[S41]
   CPLREF ANG[S41]
   CPLREF MAG[S51]   5-PORT S-PARAMETERS NOT ALLOWED
   CPLREF ANG[S51]   5-PORT S-PARAMETERS NOT ALLOWED
'REQ
   SWEEP .450 .550 .010
   STEP .500
;RID
  RANGE .4995 .5005  .0001
  GR1 0 -60 10
  GR2 180 -180 30
)PT
! optimization target forces power at isolated port of coupled line = 0
  CPLREF MAG[S31] = 0
  CPLREF MAG[S21] = 0
```

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
             MIXMODE3.OUT    Mon Feb 06 10:54:50 1995
```

FREQ-GHZ	MAG[S11] ANTENNA	ANG[S11] ANTENNA	MAG[S12] ANTENNA	ANG[S12] ANTENNA	MAG[S21] ANTENNA	ANG[S21] ANTENNA	MAG[S22] ANTENNA	ANG[S22] ANTENNA
0.45000	1.000	-1.758	0.004	119.286	0.004	119.286	1.000	60.322
0.46000	1.000	-1.798	0.005	118.691	0.005	118.691	1.000	59.171
0.47000	1.000	-1.837	0.005	118.112	0.005	118.112	1.000	58.052
0.48000	1.000	-1.876	0.005	117.551	0.005	117.551	1.000	56.969
0.49000	1.000	-1.915	0.005	117.007	0.005	117.007	1.000	55.919
0.50000	1.000	-1.955	0.005	116.478	0.005	116.478	1.000	54.899
0.51000	1.000	-1.994	0.005	115.963	0.005	115.963	1.000	53.907
0.52000	1.000	-2.033	0.005	115.464	0.005	115.464	1.000	52.947
0.53000	1.000	-2.072	0.005	114.977	0.005	114.977	1.000	52.013
0.54000	1.000	-2.112	0.006	114.504	0.006	114.504	1.000	51.104
0.55000	1.000	-2.151	0.006	114.043	0.006	114.043	1.000	50.221



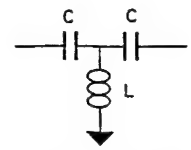
S-parameters in 50 ohm system

Y-Matrix Symmetry enforced

Y21 = Y12 = (Y12 + Y21)/2

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 10:58:21 1995

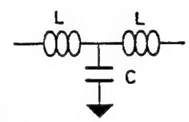
FREQ-GHZ	MAG[S11] LMN	ANG[S11] LMN	MAG[S21] LMN	ANG[S21] LMN	MAG[S22] LMN	ANG[S22] LMN
0.45000	0.988	-46.831	0.157	-136.831	0.988	-46.831
0.46000	0.985	-48.350	0.170	-138.350	0.985	-48.350
0.47000	0.983	-49.914	0.184	-139.914	0.983	-49.914
0.48000	0.980	-51.525	0.198	-141.525	0.980	-51.525
0.49000	0.977	-53.187	0.213	-143.187	0.977	-53.187
0.50000	0.973	-54.902	0.230	-144.902	0.973	-54.902
0.51000	0.969	-56.673	0.247	-146.673	0.969	-56.673
0.52000	0.964	-58.502	0.265	-148.502	0.964	-58.502
0.53000	0.959	-60.393	0.284	-150.393	0.959	-60.393
0.54000	0.953	-62.349	0.304	-152.349	0.953	-62.349
0.55000	0.946	-64.371	0.325	-154.371	0.946	-64.371



C = 2.42014 pFd
L = 9.17388 nH

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:02:08 1995

FREQ-GHZ	MAG[S11] DMN	ANG[S11] DMN	MAG[S21] DMN	ANG[S21] DMN	MAG[S22] DMN	ANG[S22] DMN
0.45000	0.999	2.235	0.047	-87.765	0.999	2.235
0.46000	0.999	2.175	0.047	-87.825	0.999	2.175
0.47000	0.999	2.117	0.046	-87.883	0.999	2.117
0.48000	0.999	2.061	0.045	-87.939	0.999	2.061
0.49000	0.999	2.007	0.044	-87.993	0.999	2.007
0.50000	0.999	1.955	0.044	-88.045	0.999	1.955
0.51000	0.999	1.904	0.043	-88.096	0.999	1.904
0.52000	0.999	1.855	0.042	-88.145	0.999	1.855
0.53000	0.999	1.807	0.042	-88.193	0.999	1.807
0.54000	0.999	1.761	0.041	-88.239	0.999	1.761
0.55000	0.999	1.716	0.041	-88.284	0.999	1.716

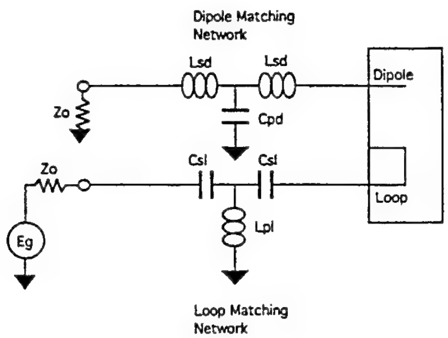


L = 408.61099 nH
C = .05459 pFd

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:06:21 1995

FREQ-GHZ	DB[S11] SPASSV	ANG[S11] SPASSV	DB[S22] SPASSV	ANG[S22] SPASSV	DB[S12] SPASSV	ANG[S12] SPASSV	DB[S21] SPASSV	ANG[S21] SPASSV
0.45000	-0.001	-13.350	-0.002	-52.961	-35.342	56.827	-35.342	56.827
0.46000	-0.004	-16.908	-0.004	-57.364	-30.623	52.842	-30.623	52.842
0.47000	-0.014	-22.942	-0.016	-64.099	-24.796	46.445	-24.796	46.445
0.48000	-0.088	-35.535	-0.092	-77.004	-16.992	33.670	-16.992	33.670
0.49000	-1.461	-76.700	-1.479	-116.882	-5.463	-6.925	-5.463	-6.925
0.50000	-47.646	-178.075	-71.352	-1.586	-0.017	-116.573	-0.017	-116.573
0.51000	-1.822	76.881	-1.848	24.829	-4.672	141.005	-4.672	141.005
0.52000	-0.170	36.926	-0.177	-13.898	-14.193	101.595	-14.193	101.595
0.53000	-0.041	24.190	-0.045	-27.034	-20.338	88.638	-20.338	88.638
0.54000	-0.015	18.070	-0.018	-33.962	-24.571	82.103	-24.571	82.103
0.55000	-0.007	14.465	-0.010	-38.548	-27.721	78.001	-27.721	78.001

Circuit for Passive S-Parameter Calculation



Lsd = 408.61099 nH

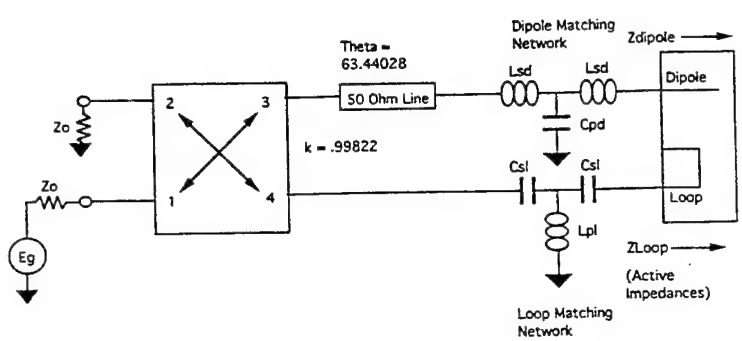
Cpd = .05459 pFd

Csl = 2.42014 pFd

Lpl = 9.17388 nH

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:09:50 1995

FREQ-GHZ	RE[S21] ZANT	IM[S21] ZANT	RE[S31] ZANT	IM[S31] ZANT
	Dipole		Loop	
0.45000	-55.077	-3.2e+03	3.618	86.747
0.46000	-59.167	-3.2e+03	3.579	88.631
0.47000	-63.387	-3.1e+03	3.537	90.535
0.48000	-67.714	-3.0e+03	3.494	92.446
0.49000	-72.184	-3.0e+03	3.449	94.366
0.50000	-74.190	-2.9e+03	3.487	96.537
0.51000	-81.179	-2.9e+03	3.365	98.250
0.52000	-85.971	-2.8e+03	3.313	100.203
0.53000	-90.718	-2.8e+03	3.264	102.172
0.54000	-95.512	-2.7e+03	3.213	104.153
0.55000	-100.328	-2.7e+03	3.161	106.146



<u>MathCad</u>

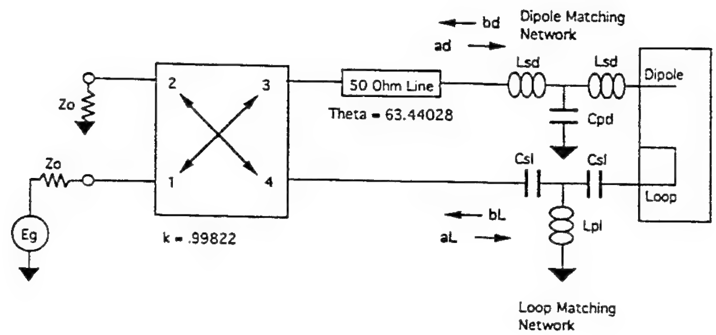
$Zactive_{dipole} = -84.045 - j\,2928.9$

$Zactive_{loop} = 3.11211 + j\,96.16673$

Differences observed

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
              MIXMODE3.OUT     Mon Feb 06 11:15:59 1995
```

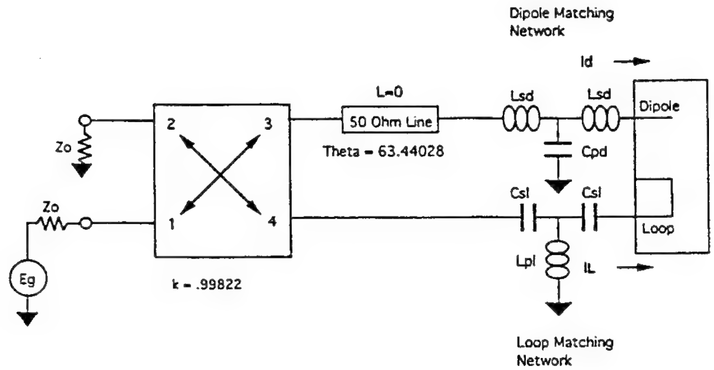
FREQ-GHZ	MAG[S21] INC	ANG[S21] INC	MAG[S31] INC	ANG[S31] INC	MAG[S21] REFL	ANG[S21] REFL	MAG[S31] REFL	ANG[S31] REFL
	a_d →		a_L →			← b_d		← b_L
0.45000	0.030	-19.264	0.030	-89.742	0.030	-32.619	0.030	-142.708
0.46000	0.029	-19.805	0.030	-84.653	0.030	-36.568	0.029	-141.872
0.47000	0.029	-20.277	0.031	-77.561	0.031	-42.526	0.029	-140.966
0.48000	0.029	-20.562	0.033	-66.130	0.033	-52.842	0.029	-139.875
0.49000	0.028	-20.081	0.043	-43.780	0.043	-74.155	0.028	-138.017
0.50000	1.198	22.075	16.748	-90.001	16.718	153.439	1.201	-94.485
0.51000	0.029	-26.012	0.046	-139.601	0.046	24.462	0.029	-141.196
0.52000	0.029	-25.569	0.034	-119.916	0.034	5.802	0.029	-139.376
0.53000	0.029	-25.900	0.032	-109.482	0.032	-3.520	0.029	-138.330
0.54000	0.029	-26.426	0.031	-102.789	0.031	-9.080	0.029	-137.478
0.55000	0.029	-27.035	0.030	-97.859	0.030	-12.870	0.029	-136.709



Note: b_L and a_d are not zero!

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
              MIXMODE3.OUT     Mon Feb 06 11:22:00 1995
```

FREQ-GHZ	MAG[S21] VANT	ANG[S21] VANT	MAG[S31] VANT	ANG[S31] VANT	MAG[S21] IANT	ANG[S21] IANT	MAG[S31] IANT	ANG[S31] IANT
	V_d			V_L		I_d		I_L
0.45000	2.422	-22.363	0.253	-125.770	7.5e-04	68.609	0.003	146.619
0.46000	2.975	-23.152	0.338	-124.447	9.4e-04	67.916	0.004	147.866
0.47000	3.909	-23.955	0.483	-123.130	0.001	67.213	0.005	149.108
0.48000	5.790	-24.782	0.776	-121.824	0.002	66.492	0.008	150.341
0.49000	11.463	-25.655	1.662	-120.560	0.004	65.731	0.018	151.534
0.50000	5.7e+03	-114.507	864.202	148.796	1.936	-23.052	8.946	60.865
0.51000	11.330	152.950	1.909	62.322	0.004	-115.430	0.019	-25.716
0.52000	5.650	152.045	1.026	63.582	0.002	-116.208	0.010	-24.524
0.53000	3.767	151.177	0.735	64.880	0.001	-116.945	0.007	-23.290
0.54000	2.833	150.309	0.593	66.186	0.001	-117.680	0.006	-22.047
0.55000	2.278	149.435	0.511	67.495	8.5e-04	-118.416	0.005	-20.799



$$\frac{I_{Loop}}{I_{dipole}} = \frac{8.846 \quad 60.865}{1.936 \quad -23.052} = 4.6209 \quad 83.917$$

previous 50 r load

$$\frac{I_L}{I_o} = \frac{0.598 \; 75.856}{0.112 \; -14.120}$$

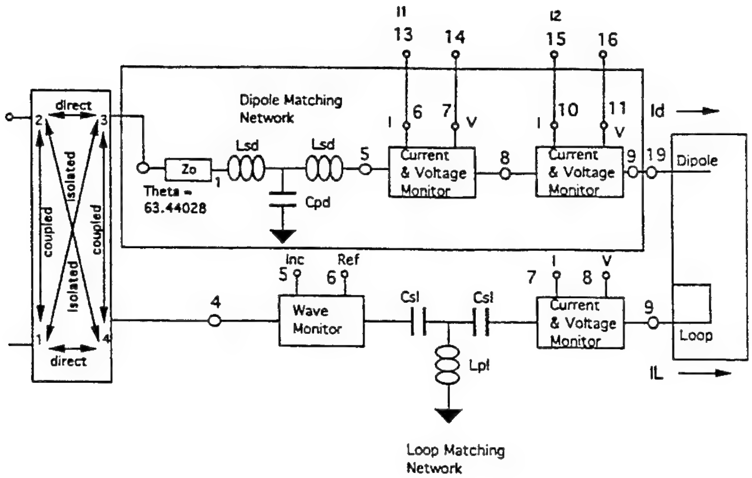
5.339 89.976

coupler 0 = 0

$$\frac{0.567 \quad 154}{0.106 \quad 64.019} = 5.349 \; 89.981$$

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:28:45 1995

FREQ-GHZ	MAG[S21]	ANG[S21]	MAG[S31]	ANG[S31]
	INORM	INORM	INORM	INORM
	Id/ Id		IL /Id	
0.45000	1.000	-4.6e-05	3.907	78.009
0.46000	1.000	-1.0e-04	4.071	79.950
0.47000	1.000	1.2e-04	4.240	81.895
0.48000	1.000	2.1e-04	4.410	83.849
0.49000	1.000	7.4e-04	4.583	85.804
0.50000	1.014	0.020	4.685	83.936
0.51000	1.000	6.6e-04	4.923	89.714
0.52000	1.000	-5.3e-04	5.106	91.683
0.53000	1.000	-1.1e-04	5.287	93.655
0.54000	1.000	-7.5e-05	5.468	95.633
0.55000	1.000	6.3e-05	5.652	97.617

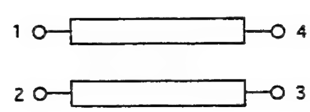


Note: $\frac{I_d}{I_d}$ should ideally always = 1 0°

resonance condition is affecting value slightly

```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
                MIXMODE3.OUT     Mon Feb 06 11:46:07 1995
```

FREQ-GHZ	MAG[S14] COUPLER	ANG[S14] COUPLER	MAG[S41] COUPLER	ANG[S41] COUPLER	MAG[S23] COUPLER	ANG[S23] COUPLER	MAG[S32] COUPLER	ANG[S32] COUPLER
0.45000	0.060	-89.459	0.060	-89.459	0.060	-89.459	0.060	-89.459
0.46000	0.060	-89.568	0.060	-89.568	0.060	-89.568	0.060	-89.568
0.47000	0.060	-89.677	0.060	-89.677	0.060	-89.677	0.060	-89.677
0.48000	0.060	-89.785	0.060	-89.785	0.060	-89.785	0.060	-89.785
0.49000	0.060	-89.893	0.060	-89.893	0.060	-89.893	0.060	-89.893
0.50000	0.060	-90.000	0.060	-90.000	0.060	-90.000	0.060	-90.000
0.51000	0.060	-90.107	0.060	-90.107	0.060	-90.107	0.060	-90.107
0.52000	0.060	-90.215	0.060	-90.215	0.060	-90.215	0.060	-90.215
0.53000	0.060	-90.323	0.060	-90.323	0.060	-90.323	0.060	-90.323
0.54000	0.060	-90.432	0.060	-90.432	0.060	-90.432	0.060	-90.432
0.55000	0.060	-90.541	0.060	-90.541	0.060	-90.541	0.060	-90.541



$$\begin{bmatrix} 0 & 0.998-180 & 0 & 0.060-90 \\ 0.998-180 & 0 & 0.060-90 & 0 \\ 0 & 0.060-90 & 0 & 0.998-180 \\ 0.060-90 & 0 & 0.998-180 & 0 \end{bmatrix}$$

$$k = 0.99822$$

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:47:06 1995

FREQ-GHZ	MAG[S13] COUPLER	ANG[S13] COUPLER	MAG[S31] COUPLER	ANG[S31] COUPLER	MAG[S24] COUPLER	ANG[S24] COUPLER	MAG[S42] COUPLER	ANG[S42] COUPLER
0.45000	2.9e-11	0.000	2.9e-11	0.000	2.9e-11	0.000	2.9e-11	0.000
0.46000	1.9e-09	90.448	1.9e-09	90.448	1.9e-09	90.448	1.9e-09	90.448
0.47000	1.9e-09	-90.000	1.9e-09	-90.000	1.9e-09	-90.000	1.9e-09	-90.000
0.48000	7.3e-12	0.000	7.3e-12	0.000	7.3e-12	0.000	7.3e-12	0.000
0.49000	1.9e-09	-89.888	1.9e-09	-89.888	1.9e-09	-89.888	1.9e-09	-89.888
0.50000	1.9e-09	-90.000	1.9e-09	-90.000	1.9e-09	-90.000	1.9e-09	-90.000
0.51000	1.9e-09	-90.112	1.9e-09	-90.112	1.9e-09	-90.112	1.9e-09	-90.112
0.52000	7.3e-12	180.000	7.3e-12	180.000	7.3e-12	180.000	7.3e-12	180.000
0.53000	1.9e-09	-89.105	1.9e-09	-89.105	1.9e-09	-89.105	1.9e-09	-89.105
0.54000	1.5e-11	0.000	1.5e-11	0.000	1.5e-11	0.000	1.5e-11	0.000
0.55000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:48:14 1995

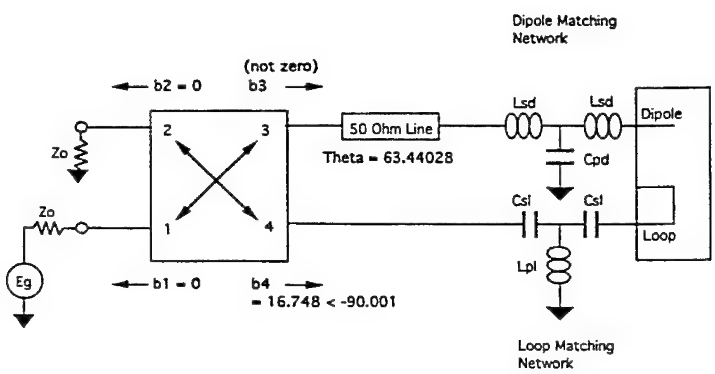
FREQ-GHZ	MAG[S11] COUPLER	ANG[S11] COUPLER	MAG[S22] COUPLER	ANG[S22] COUPLER	MAG[S33] COUPLER	ANG[S33] COUPLER	MAG[S44] COUPLER	ANG[S44] COUPLER
0.45000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.46000	3.0e-08	0.448	3.0e-08	0.448	3.0e-08	0.448	3.0e-08	0.448
0.47000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000
0.48000	6.0e-08	-179.888	6.0e-08	-179.888	6.0e-08	-179.888	6.0e-08	-179.888
0.49000	1.2e-10	-90.000	1.2e-10	-90.000	1.2e-10	-90.000	1.2e-10	-90.000
0.50000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000
0.51000	5.8e-11	90.000	5.8e-11	90.000	5.8e-11	90.000	5.8e-11	90.000
0.52000	6.0e-08	179.776	6.0e-08	179.776	6.0e-08	179.776	6.0e-08	179.776
0.53000	3.0e-08	-179.552	3.0e-08	-179.552	3.0e-08	-179.552	3.0e-08	-179.552
0.54000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000	3.0e-08	180.000
0.55000	4.7e-10	-90.000	4.7e-10	-90.000	4.7e-10	-90.000	4.7e-10	-90.000

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:49:22 1995

FREQ-GHZ	MAG[S12] COUPLER	ANG[S12] COUPLER	MAG[S21] COUPLER	ANG[S21] COUPLER	MAG[S34] COUPLER	ANG[S34] COUPLER	MAG[S43] COUPLER	ANG[COU
0.45000	0.998	-179.459	0.998	-179.459	0.998	-179.459	0.998	-179
0.46000	0.998	-179.568	0.998	-179.568	0.998	-179.568	0.998	-179
0.47000	0.998	-179.677	0.998	-179.677	0.998	-179.677	0.998	-179
0.48000	0.998	-179.785	0.998	-179.785	0.998	-179.785	0.998	-179
0.49000	0.998	-179.893	0.998	-179.893	0.998	-179.893	0.998	-179
0.50000	0.998	-180.000	0.998	-180.000	0.998	-180.000	0.998	-180
0.51000	0.998	179.893	0.998	179.893	0.998	179.893	0.998	179
0.52000	0.998	179.785	0.998	179.785	0.998	179.785	0.998	179
0.53000	0.998	179.677	0.998	179.677	0.998	179.677	0.998	179
0.54000	0.998	179.568	0.998	179.568	0.998	179.568	0.998	179
0.55000	0.998	179.459	0.998	179.459	0.998	179.459	0.998	179

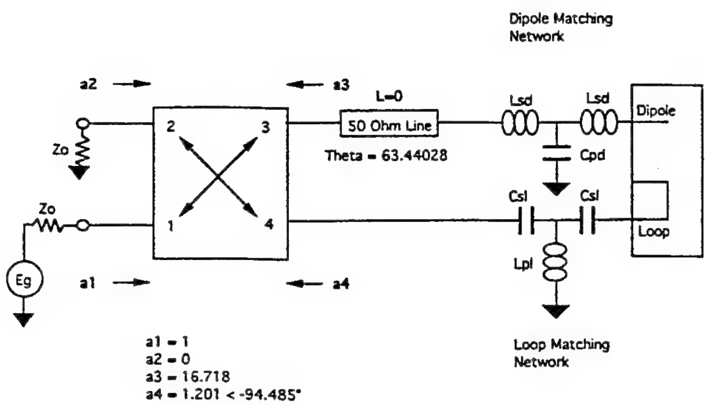
Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:58:15 1995

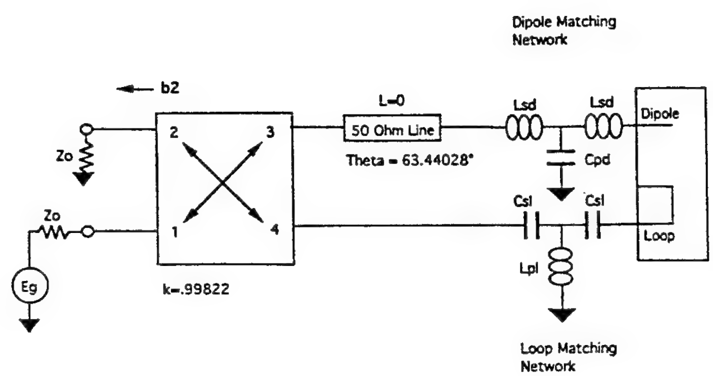
FREQ-GHZ	DB[S21] CPLREF	ANG[S21] CPLREF	DB[S31] CPLREF	ANG[S31] CPLREF	MAG[S41] CPLREF	ANG[S41] CPLREF	AG[S31] INC	ANG[S31] INC
	b1		b2			b3		aL →
0.45000	-54.918	127.835	-1.4e-05	-179.458	0.030	37.833	0.030	-89.742
0.46000	-55.070	128.561	-1.4e-05	-179.577	0.029	38.560	0.030	-84.653
0.47000	-55.200	129.360	-1.4e-05	-179.699	0.029	39.357	0.031	-77.561
0.48000	-55.319	130.344	-1.7e-05	-179.830	0.029	40.340	0.033	-66.130
0.49000	-55.469	132.093	-3.0e-05	-179.999	0.028	42.091	0.043	-43.780
0.50000	-22.902	175.515	-58.715	-178.790	1.198	85.515	16.748	-90.001
0.51000	-55.141	128.699	-3.8e-05	-179.987	0.029	38.697	0.046	-139.601
0.52000	-55.218	130.412	-2.1e-05	179.844	0.029	40.409	0.034	-119.916
0.53000	-55.203	131.349	-1.6e-05	179.713	0.029	41.347	0.032	-109.482
0.54000	-55.145	132.092	-1.6e-05	179.591	0.029	42.090	0.031	-102.789
0.55000	-55.053	132.751	-1.6e-05	179.472	0.029	42.750	0.030	-97.859

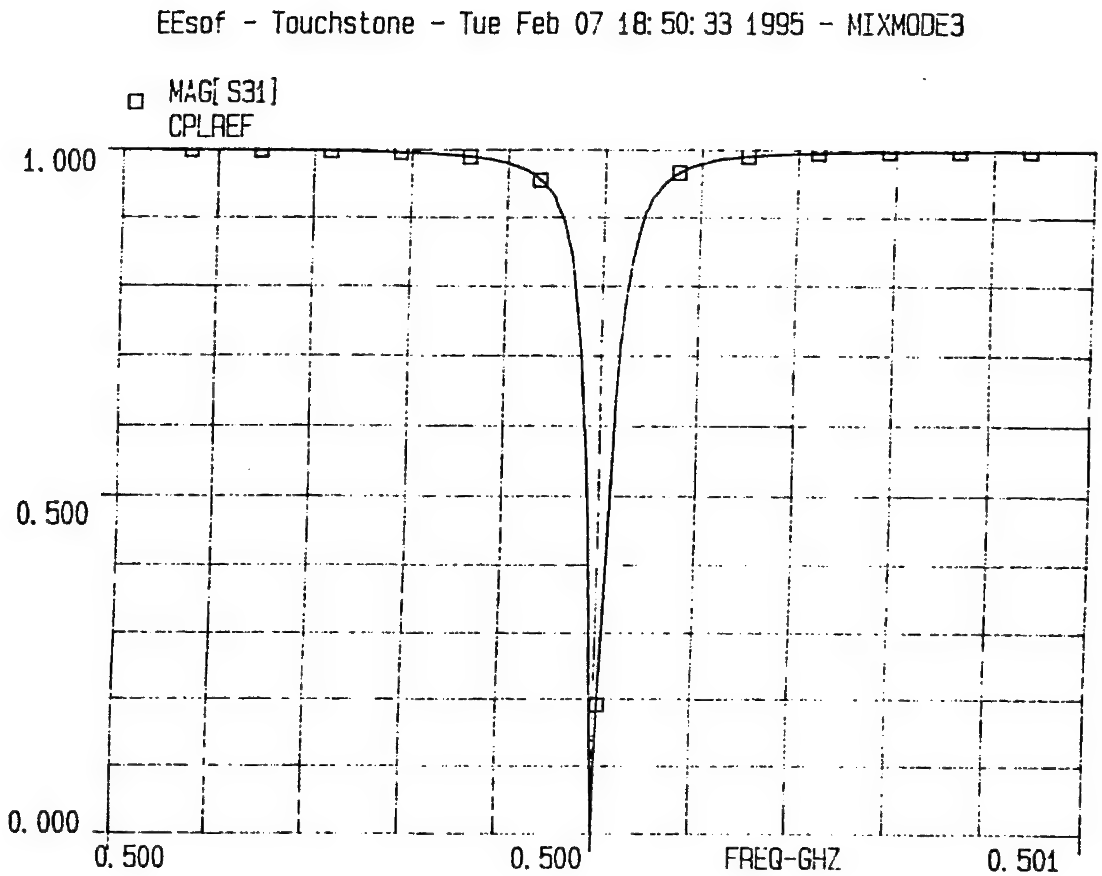


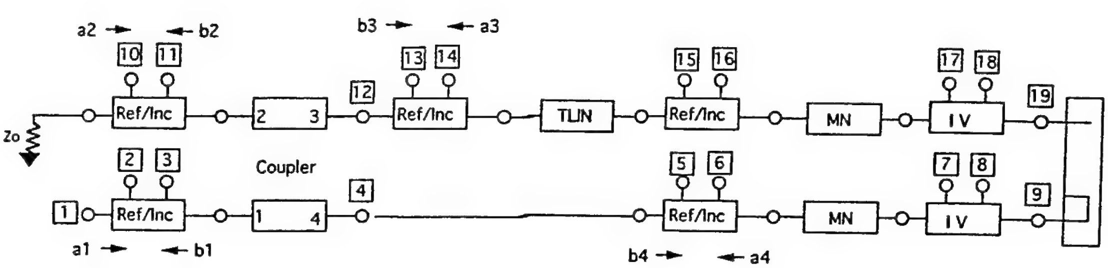
Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 11:57:08 1995

FREQ-GHZ	MAG[S21] CPLINC	ANG[S21] CPLINC	MAG[S31] CPLINC	ANG[S31] CPLINC	MAG[S41] CPLINC	ANG[S41] CPLINC	MAG[S31] REFL	ANG[S31] REFL
		$a_1 \rightarrow$		$a_2 \rightarrow$	$\leftarrow a_3$			$\leftarrow b_L$
0.45000	1.000	0.000	7.1e-08	0.542	0.030	-89.715	0.030	-142.708
0.46000	1.000	0.000	7.1e-08	0.423	0.030	-94.933	0.029	-141.872
0.47000	1.000	0.000	7.1e-08	0.301	0.031	-102.160	0.029	-140.966
0.48000	1.000	0.000	7.1e-08	0.170	0.033	-113.745	0.029	-139.875
0.49000	1.000	0.000	7.1e-08	7.6e-04	0.043	-136.327	0.028	-138.017
0.50000	1.000	0.000	8.3e-11	1.210	16.718	89.999	1.201	-94.485
0.51000	1.000	0.000	7.1e-08	0.013	0.046	-40.247	0.029	-141.196
0.52000	1.000	0.000	7.1e-08	-0.156	0.034	-60.176	0.029	-139.376
0.53000	1.000	0.000	7.1e-08	-0.287	0.032	-70.766	0.029	-138.330
0.54000	1.000	0.000	7.1e-08	-0.409	0.031	-77.595	0.029	-137.478
0.55000	1.000	0.000	7.1e-08	-0.528	0.030	-82.654	0.029	-136.709









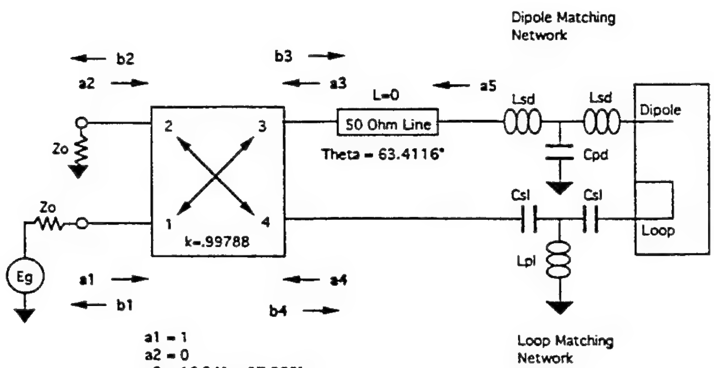
2	a1	3	b1
10	a2	4	b2
14	a3	13	b3
6	a4	5	b4

Appendix I

TOUCHSTONE ANALYSIS OF MIXED-MODE ARRAY (OPTIMUM FEEDBACK - DETERMINED BY MATHCAD)

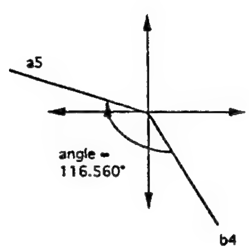
```
Touchstone (TM) - Configuration( 100 1600 100 15713 1604 1000 1 3294 )
                  MIXMODE3.OUT    Mon Feb 06 18:08:23 1995
```

FREQ-GHZ	MAG[S21] CPLINC	ANG[S21] CPLINC	MAG[S31] CPLINC	ANG[S31] CPLINC	MAG[S41] CPLINC	ANG[S41] CPLINC
		a1		a2		a3
0.45000	1.000	0.000	7.1e-08	0.591	0.033	-89.641
0.46000	1.000	0.000	7.1e-08	0.461	0.033	-94.871
0.47000	1.000	0.000	7.1e-08	0.326	0.033	-102.111
0.48000	1.000	0.000	7.1e-08	0.181	0.036	-113.710
0.49000	1.000	0.000	7.1e-08	-0.010	0.047	-136.314
0.50000	1.000	0.000	1.0e-08	-113.051	16.341	97.250
0.51000	1.000	0.000	7.1e-08	0.026	0.050	-40.217
0.52000	1.000	0.000	7.1e-08	-0.165	0.038	-60.158
0.53000	1.000	0.000	7.1e-08	-0.310	0.035	-70.759
0.54000	1.000	0.000	7.1e-08	-0.444	0.034	-77.599
0.55000	1.000	0.000	7.1e-08	-0.575	0.033	-82.669



a1 = 1
a2 = 0
a3 = 16.341 < 97.250°
a4 = 1.076 < -80.020°
a5 = 16.341<160.661°

b1 = .070
b2 = .146
b3 = 1.074 < 99.980°
b4 = 16.371 < -82.779°



$$20 \log \left(\frac{b_3}{a_3}\right) = -23.6 \text{ dB}$$

$$20 \log \left(\frac{a_4}{b_4}\right) = -23.65 \text{ dB}$$

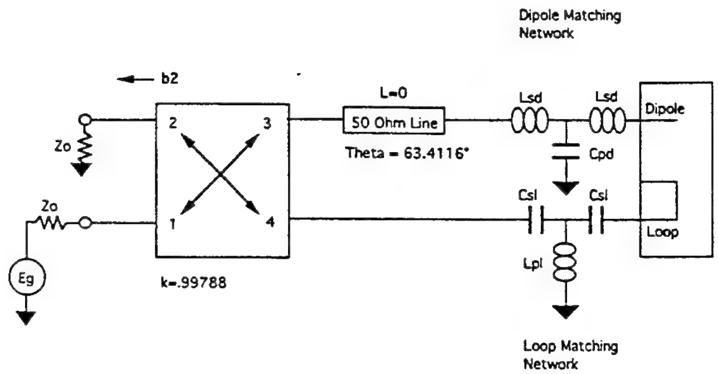
$$20 \log \left(\frac{b_1}{a_1}\right) = -23.092 \text{ dB}$$

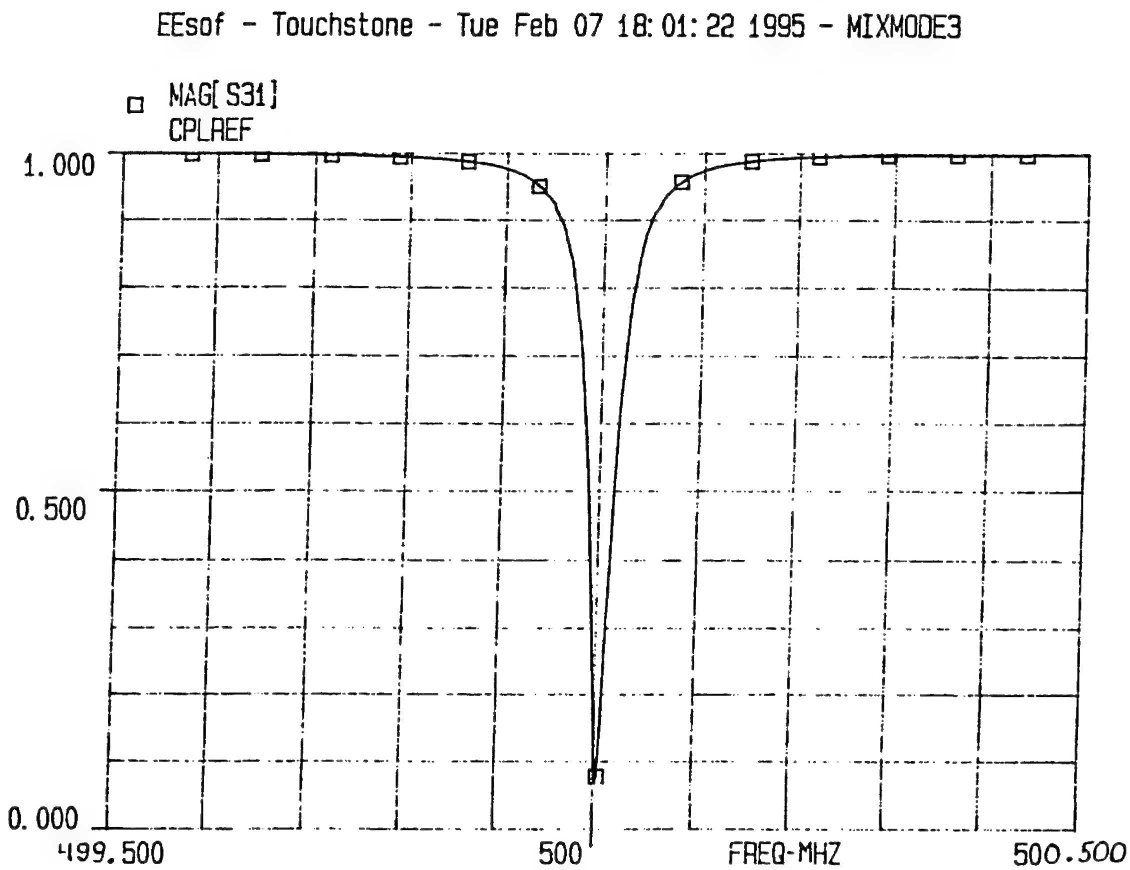
Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 18:09:33 1995

FREQ-GHZ	DB[S21] CPLREF	ANG[S21] CPLREF	DB[S31] CPLREF	ANG[S31] CPLREF	MAG[S41] CPLREF	ANG[S41] CPLREF	
	b1		b2		b3		b4
0.45000	-53.398	127.859	-2.0e-05	-179.409	0.032	37.857	
0.46000	-53.550	128.574	-2.0e-05	-179.539	0.032	38.572	
0.47000	-53.682	129.357	-2.1e-05	-179.674	0.032	39.356	
0.48000	-53.802	130.324	-2.3e-05	-179.819	0.031	40.324	
0.49000	-53.955	132.053	-3.8e-05	179.990	0.031	42.052	
0.50000	-23.092	-170.020	-16.722	66.949	1.074	99.980	16.371 < -82.779
0.51000	-53.617	128.682	-4.8e-05	-179.974	0.032	38.681	
0.52000	-53.698	130.371	-2.7e-05	179.835	0.032	40.371	
0.53000	-53.684	131.295	-2.4e-05	179.690	0.032	41.294	
0.54000	-53.626	132.026	-2.1e-05	179.556	0.032	42.024	
0.55000	-53.534	132.673	-2.1e-05	179.425	0.032	42.671	

Touchstone (TM) - Configuration(100 1600 100 15713 1604 1000 1 3294)
MIXMODE3.OUT Mon Feb 06 18:07:27 1995

FREQ-GHZ	MAG[S21] INC	ANG[S21] INC	MAG[S31] INC	ANG[S31] INC	MAG[S21] REFL	ANG[S21] REFL	MAG[S31] REFL	ANG[S31] REFL
				b4		a5		a4
0.45000	0.032	-19.214	0.033	-89.767	0.033	-32.571	0.032	-142.734
0.46000	0.032	-19.767	0.033	-84.679	0.033	-36.532	0.032	-141.899
0.47000	0.032	-20.251	0.033	-77.589	0.033	-42.504	0.032	-140.997
0.48000	0.031	-20.551	0.036	-66.160	0.036	-52.835	0.031	-139.911
0.49000	0.031	-20.091	0.047	-43.812	0.047	-74.171	0.031	-138.065
0.50000	1.074	36.568	16.371	-82.779	16.341	160.661	1.076	-80.020
0.51000	0.032	-25.999	0.050	-139.609	0.050	24.463	0.032	-141.202
0.52000	0.032	-25.577	0.038	-119.935	0.038	5.790	0.032	-139.394
0.53000	0.032	-25.922	0.035	-109.506	0.035	-3.543	0.032	-138.354
0.54000	0.032	-26.461	0.034	-102.817	0.034	-9.114	0.032	-137.505
0.55000	0.032	-27.082	0.033	-97.889	0.033	-12.916	0.032	-136.738

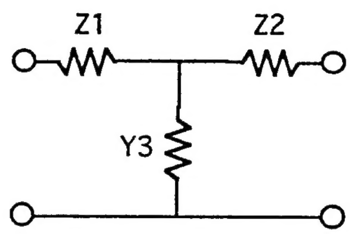

Dipole Matching
Network
b2
L=0
50 Ohm Line
Theta = 63.4116°
Lsd
Lsd
Dipole
Cpd
Csl
Csl
Loop
Lpl
Zo
Zo
Eg
2
3
1
4
k=.99788
Loop Matching
Network


EEsof - Touchstone - Tue Feb 07 18:01:22 1995 - MIXMODE3
MAG[S31]
CPLREF
1.000
0.500
0.000
499.500
500
FREQ-MHZ
500.500

Appendix J

DERIVATION OF Z-MATRIX FOR A TEE-SECTION

ABCD of a Tee Network



$$\begin{bmatrix} 1+Z_1Y & (1+Z_1Y)Z_2+Z_1 \\ Y & YZ_2+1 \end{bmatrix} = ABCD\ Tee$$

Convert back to Z matrix

$$Z_{11} = \frac{A}{C} \qquad Z_{22} = \frac{D}{C}$$

$$Z_{12} = Z_{21} = \frac{1}{C}$$

$$\begin{bmatrix} \frac{1}{Y}+Z_1 & \frac{1}{Y} \\ \frac{1}{Y} & \frac{1}{Y}+Z_2 \end{bmatrix} = Z_{TEE}$$

let

$$Z_3 = \frac{1}{Y}$$

$$Z_{11} = Z_1 + Z_3 \qquad Z_{12} = Z_{21} = Z_3$$

$$Z_{22} = Z_2 + Z_3$$

So

$$Z_{11} = Z_1 + Z_{12} \qquad \text{or} \qquad Z_1 = Z_{11} - Z_{12}$$

$$Z_{22} = Z_2 + Z_{12} \qquad \text{or} \qquad Z_2 = Z_{22} - Z_{12}$$

$$Z_3 = Z_{12}$$

$$Z_{TEE}\begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$

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